Introduction

The coherence attribute has proven to be a very useful attribute for highlighting structural and stratigraphic discontinuities such as faults, fractures and channels in 3-D seismic volumes. The coherence attribute was first proposed by Bahorich and Farmer (1995), and it was based on normalized cross-correlation between each trace and its inline and crossline neighbors. Marfurt et al. (1998) then proposed a multi-trace semblance-based coherence algorithm that was more robust to noise and improved vertical resolution. Later, Gersztenkorn and Marfurt (1999) proposed an improved coherence algorithm, called C3 coherence, which is based on the eigenstructure of covariance matrices of windowed seismic traces. Recently, there has been renewed interest in the coherence attribute. Yang et al. (2015) proposed a computationally efficient coherence algorithm based on a normalized information divergence criterion that avoids directly calculating the eigenvalues of the covariance matrix. In addition, Li and Lu (2014) combined spectral decomposition and complex coherence computation to map discontinuities at different scales. Finally, in order to avoid false low-coherence values in steeply dipping structures, Sui et al. (2015) proposed a coherence algorithm that analyses the eigenstructure of the spectral amplitudes of seismic traces.

The C3 coherence is based on the eigenstructure of the covariance matrix of the zero-mean traces in the analysis cube. We will show that this is analogous to unfolding a 3rd-order analysis tensor in a single mode and computing the covariance matrix of that mode. By unfolding the tensor along the other two modes, and repeating the process, then assigning each coherence attribute from each mode a different color we can greatly enhance the amount of detail that the C3 coherence can extract from seismic volumes.

In this paper, our contributions are two-fold. Firstly, we extend the coherence attribute as presented in Gersztenkorn and Marfurt (1999) to a generalized tensor-based coherence (GTC) attribute that greatly enhances the details, and allows more fine-tuning and flexibility to the interpreter. Secondly, we present a novel preprocessing step for the coherence attribute using a multivariate Gaussian kernel. This preprocessing step allows the coherence algorithm to weigh different traces in the analysis tensor according to their physical distance from the reference trace. This enhances the results of the coherence attribute that we propose, and increases its robustness to noise. This new tensor-based coherence can be viewed as a generalization of the C3 coherence attribute that was proposed by Gersztenkorn and Marfurt (1999).

Generalized Tensor-Based Coherence (GTC) Algorithm

Given a migrated 3D seismic volume, the coherence attribute for each voxel in the volume is computed within a small 3D analysis cuboid of size $I_1 \times I_2 \times I_3$. The subscripts 1, 2 and 3 throughout this section refer to the dimensions along time (or depth), inline, and crossline respectively. Each analysis cuboid can be represented as a $3^{nd}$ order tensor $A \in \mathbb{R}^{I_1 \times I_2 \times I_3}$ that we refer to as the analysis tensor. To compute the covariance matrices of this tensor, we unfold the tensor along its three modes. In general, mode-$n$ unfolding of an $N$-th order tensor results in a matrix $A_{(n)}$ of size $I_n$ by $(I_1 \cdots I_{n-1} I_{n+1} \cdots I_N)$ where the tensor element indexed by $(i_1, i_2, \cdots, i_N)$ now corresponds to the element $(i_n, j)$ in $A_{(n)}$ where

$$ j = 1 + \sum_{k=1}^{N} (i_k - 1) \prod_{m=1, m\neq n}^{k-1} I_m. \quad (1) $$

For additional details, see Aja-Fernández et al. (2009). Thus unfolding the tensor along its three modes results in three matrices: the $I_1 \times I_2 I_3$ mode-1 matrix $A_{(1)}$ unfolded along the time (depth) dimension, the $I_2 \times I_1 I_3$ mode-2 matrix $A_{(2)}$ unfolded along the inline dimension, and the $I_3 \times I_1 I_2$ mode-3 matrix $A_{(3)}$ unfolded along the crossline dimension. The covariance matrices are then given by

$$ C_1 = (A_{(1)} - 1_{I_1 \times 1} \mu_1)^T (A_{(1)} - 1_{I_1 \times 1} \mu_1), \quad (2) $$
where $\mu_1$ is a row vector of length $I_2I_3$ containing the means of all columns of $A_{(1)}$, and $1_{I_1 \times 1}$ is a column vector of ones of length $I_1$. $C_3$ and $C_3$ are also computed in a similar fashion. These covariance matrices are positive semi-definite matrices and thus all their eigenvalues are non-negative. If we denote the ranked eigenvalues of $C_1$ as $\lambda^{(1)}_1 = \lambda^{(1)}_1, \lambda^{(1)}_2, \cdots, \lambda^{(1)}_{I_2I_3}$, and similarly for $C_2$ and $C_3$, then the coherence attributes of the three different modes are given as the ratios of the largest eigenvalue of the covariance matrix to its trace. Specifically,

$$E_c^{(1)} = \frac{\lambda^{(1)}_1}{Tr(C_1)}, \quad E_c^{(2)} = \frac{\lambda^{(2)}_1}{Tr(C_2)}, \quad E_c^{(3)} = \frac{\lambda^{(3)}_1}{Tr(C_3)}.$$

Here, $E_c^{(1)}$ corresponds to the C3 coherence attribute that was proposed by Gersztenkorn and Marfurt (1999). By combining the C3 attribute ($E_c^{(1)}$) with the coherence estimates of the analysis tensor unfolded along mode-2 ($E_c^{(2)}$) and mode-3 ($E_c^{(3)}$) in different color channels, we arrive at the proposed coherence attribute that we refer to as the GTC coherence attribute. This way, we can add richer details to the coherence volume while preserving the original values of the C3 attribute in a single color channel.

**Preprocessing with a multivariate Gaussian kernel**

Most coherence algorithms treat all traces in an analysis tensor equally regardless of their proximity to the reference trace. This is understandable if the analysis tensor dimensions were very small. However, as the dimensions of the tensor become larger, this introduces noise to the different covariance matrices. This is the case when either or all the dimensions $I_1$, $I_2$, and $I_3$ are greater than or equal to 5. The proposed preprocessing step weighs different traces by weights relative to their proximity to the reference trace, and thus eliminates this problem.

Given the 3-D analysis tensor $A$, we can preprocess it by taking its Hadamard product (Horn and Johnson (1986)) with a 3-dimensional Gaussian kernel of the same size as $A$. We can write the preprocessed analysis tensor as

$$\tilde{A} = A \odot \mathcal{G},$$

where $\odot$ is the Hadamard product, and $\mathcal{G}$ is the multivariate Gaussian kernel given by

$$\mathcal{G}(x; \mu, \Sigma) = e^{-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)}.$$

Here, $x = \{z, x, y\}$ represents the voxels in the seismic volume, $\mu = \{z_0, x_0, y_0\}$ is the reference voxel, and $\Sigma$ is the $3 \times 3$ covariance matrix of $\mathcal{G}$. The expression in equation (7) is equivalent to a multivariate Gaussian distribution multiplied by $(2\pi)^{\frac{3}{2}} |\Sigma|$. The values of $\Sigma$ describe the shape of the multivariate Gaussian kernel. The interpreter can select these values to give more emphasis to the coherence attribute extracted along any unfolding mode, or any combination of unfolding modes. Extracting the GTC coherence attribute from $\tilde{A}$ as opposed to $A$ greatly enhances the results.

**Results**

In this section, we apply the proposed GTC attribute, as well as the preprocessing method, on the Netherlands offshore F3 block in the North Sea provided by dGB Earth Sciences (1987). As an example, Fig.1 shows the result of applying the C3 (or $E_c^{(1)}$) coherence attribute and the proposed GTC coherence attribute on the 952 ms time section of the Netherlands offshore F3 block. The red arrows in the figure indicate three different channels. They are more easily distinguished using the GTC attribute. In addition,
Figure 1: Results of applying the C3 (or $E^{(1)}_c$) attribute and the proposed GTC attribute on the 952 ms time section of the Netherlands offshore F3 block. In Fig. 1c, $E^{(1)}_c$ is assigned the blue channel, $E^{(2)}_c$ the green channel, and $E^{(3)}_c$ the red channel.
other subtle features are more clearly visible in the GTC coherence image. Adjusting the dimensions of the analysis tensor $I_1, I_2$ and $I_3$ or adjusting the values in the covariance matrix of the multivariate Gaussian kernel $\Sigma$ allows the interpreter more freedom and flexibility with the coherence attribute. For brevity, we restrict ourselves here to the special case when $\Sigma = \text{diag}\{\sigma, \sigma, \sigma\}$, that is, when the variances of the multivariate Gaussian kernel along all the dimensions are the same, and the covariances are all zero. In Fig. 2 we show the effect of varying the value of $\sigma$ for this case. As the values of $\sigma$ gets larger, $\mathcal{A}$ approaches $\mathcal{A}$, and the effect of the preprocessing step diminishes. Another special case would be when $\Sigma = \text{diag}\{\infty, 0, 0\}$, in this case the GTC coherence of $\mathcal{A}$ would simply be the C3 coherence.

Conclusions

In conclusion, we have proposed a generalized tensor-based coherence attribute. We have demonstrated that this attribute is a generalization of the C3 attribute along different unfolding modes of the analysis tensor. We have also presented a preprocessing method that weighs the traces in the analysis tensor based on their proximity to the reference trace. We show that this preprocessing step allows interpreters more flexibility, while enhancing the results at the same time. Results from the Netherlands F3 offshore block show the effectiveness of this attribute compared to C3 coherence attribute.

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References


