Using the Coefficient of Variation to Improve the Sparsity of Seismic Data

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***Abstract*—In this work we propose using the coefficient of variation as a cost function to improve seismic data representation in the curvelet domain. Performance improvement is demonstrated in denoising and compressed sensing data recovery. The demonstrated approach can be extended to other seismic applications and alternate transforms.**

***Index Terms*—Sparsity, Curvelet, Compressed Sensing**

I. Introduction

Directional transforms are gaining popularity in a variety of different applications. In the seismic domain improved performance is reported in areas like denoising, multiple reduction, migration, and fault detection. The Curvelet transform provides one of the most efficient sparse representation of seismic data. The coefficients are generated by taking the inverse fourier transfrom of directional tiles arranged to cover the frequency domain (Fig. 1). Recently, we proposed adapting curvelet tilings to better represent the FFT content in the class of images of interest [1]. Deionising performance, measured in MSE or PSNR, was used as a cost function. Nelder-Mead derivative free method is used to find the optimal tiling locations. In this work we show that improved distribution of curvelet tiles can be obtained by maximizing the coefficient of variation (C_v), where $C_v = \frac{\sigma}{\mu}$, of curvelet coefficients magnitude.

II. Sparsity measures

In addition to the coefficient of variation, we tested using shannon's entropy and the Gini index as measures guiding the adaptation process. Shannon's entropy was used by Coifman *et. al.* [2] to find the best wavelet basis in wavelet packets. The Gini index is a measure of economic statistical dispersion that was recommended for use as a measure of data sparsity in [3]. Note that since curvelet tiling adaptations alter the length of the coefficients vector, minimizing l^p norms will not generate satisfactory results.

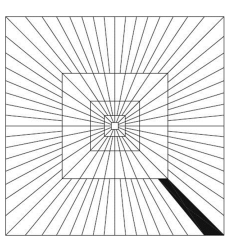

Fig. 1. Default curvelet tiling

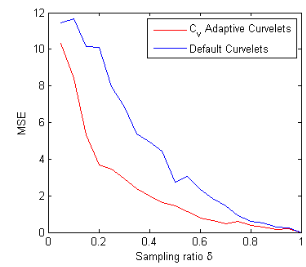

Fig. 2. Seismic data recovery from incomplete measurements

III. Results

A Gaussian noise curvelet denoising algorithm was used to compare the different sparsity measures. Results are summarized in table 1. Next, the C_v optimizer was used to improve seismic data recovery from missing traces. Mathematically, the problem can be formulated as follows [4]:

$$\hat{\mathbf{f}} = C^{-1}(\hat{\mathbf{x}}) \quad \text{with} \quad \hat{\mathbf{x}} = \arg_{\mathbf{x}} \min \|\mathbf{x}\|_1 \quad \text{subject to } SC^{-1}(\mathbf{x}) = \mathbf{b} \tag{1}$$

Where S is the sampling matrix that randomly samples $\delta = \frac{n}{N}$ seismic traces. $C^{-1}(\mathbf{x})$ is the inverse curvelet transform of $\mathbf{x}$. The SPGL1 solver was used for data recovery. Fig. 2 summarizes the results obtained.

TABLE I
Average Seismic denoising performance comparison between default and adaptive tiling curvelets. Five seismic data sets of size 550×100 were used in this experiemnt.

	Denoising MSE, $\sigma = 1$			
	Default Curvelets	Adaptive Curvelets		
		Entropy	Gini index	C_v
Avg. MSE	0.8230	0.9371	0.6102	**0.4766**

References

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