

Energy-Constrained Distributed Estimation in Wireless Sensor Networks

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Abstract—We consider distributed parameter estimation in energy-constrained wireless sensor networks, where limited energy is allowed to be used by all sensors at each task period. Thus there exists a tradeoff between the number of active sensors and the energy used by each active sensor to minimize the estimation MSE. To determine the optimal energy scheduling of each sensor, a concept of the equivalent unit-energy MSE function is introduced. Based on this concept, an optimal distributed estimation algorithm for homogeneous sensor networks and a quasi-optimal distributed estimation algorithm for heterogeneous sensor networks are proposed. Moreover, a theoretical non-achievable lower bound of the estimation MSE under the total energy constraint is proved and it is shown that our proposed algorithm is quasi-optimal and within a factor 2 of the theoretical lower bound. Simulation results also show that a significant reduction in the estimation MSE is achieved by the proposed method when compared with other uniform schemes.

I. INTRODUCTION

A wireless sensor network (WSN) is composed of a large number of geographically distributed sensor nodes. Though each sensor is characterized by low power constraint and limited computation and communication capabilities, potentially powerful networks can be constructed to accomplish various high-level tasks via sensor cooperation [1], such as distributed estimation, distributed detection, and target localization and tracking.

Distributed Estimation of unknown deterministic parameters by a set of distributed sensor nodes and a fusion center (FC) has become an important topic in signal processing research for sensor networks. Subject to severe bandwidth and energy constraints, each sensor is allowed to transmit only a quantized version of its raw measurement to the fusion center (FC) that generates a final parameter estimation. Recently, several bandwidth-constrained distributed estimation algorithms have been investigated [2]–[6]. The work of [2] addressed various

design and implementation issues to digitize the transmitted signal into one or several binary bits using the joint distribution of sensors' data. In [3], a class of maximum likelihood estimators (MLE) was proposed to attain a variance that is close to the clairvoyant estimator when the observations are quantized to one bit. Without the knowledge of noise distribution, the work of [4] and [5] proposed several universal (pdf-unaware) decentralized estimation systems based on best linear unbiased estimation (BLUE) rule for distributed parameter estimation in the presence of unknown, additive sensor noise. In [6], we proposed quasi-optimal distributed parameter estimation algorithms to minimize the estimation mean square error (MSE) under a total rate constraint. On the other hand, the minimal-energy distributed estimation problem has been recently considered in [7] and [8], where the total sensor transmission energy is minimized by selecting the optimal quantization levels while meeting the target estimation MSE requirements.

In this paper, we address the distributed parameter estimation for energy-constrained wireless sensor networks. More specifically, only limited amount of energy, namely P_c , is allowed to be consumed by all sensors at each task period, and thus needs to be optimally allocated among all sensors to minimize the estimation MSE. Based on this concept, there exists an interesting tradeoff between the number of active sensors and allocated energy at each active sensor. We solve this optimal tradeoff and design the distributed estimation algorithm by: (i) selecting a subset of active sensors to observe the phenomenon, and (ii) determining the energy allocated at each active sensor to transmit its quantized message (locally processed observation) to the fusion center, which performs the final estimation.

The rest of the paper is organized as follows. Section II states the distributed estimation problem under the total energy constraint. Section III introduces a concept of equivalent unit-energy MSE function. Then in Section IV

and section V, we propose optimal and quasi-optimal distributed estimation algorithms for homogeneous and heterogeneous sensor networks, respectively. Section VI summarizes some simulation results that demonstrate the performance of the proposed algorithms. Finally, conclusions are given in Section VII.

II. PRELIMINARY AND PROBLEM STATEMENT

We consider a dense sensor network that includes N distributed sensors and a fusion center to estimate the unknown parameter θ . Since the total energy allowed for all sensors is limited, it implies a tradeoff between the number of active sensors and the energy used at each active sensor. The distributed estimation system can be described as follows (Fig. 1). First, we assume K active sensors (i_1, \dots, i_K), and each active sensor makes an observation, which is corrupted by spatially uncorrelated additive noises and is described by $x_{i_k} = \theta + n_{i_k}$. Second, each of the K active sensors performs a local quantization $\hat{x}_{i_k} = Q(x_{i_k})$ and transmits its quantized message to the fusion center which produces a final estimation of θ .

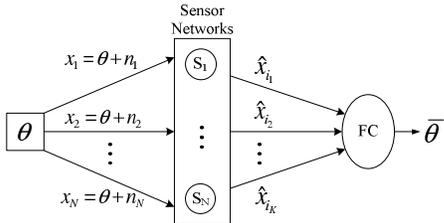


Fig. 1. Distributed estimation system.

If the fusion center has the knowledge of the sensor noise variance $\sigma_{i_k}^2$ and the sensors can perfectly send their observations x_{i_k} to the fusion center, the BLUE estimator [9] for θ is known to be

$$\bar{\theta} = \left(\sum_{k=1}^K \frac{1}{\sigma_{i_k}^2} \right)^{-1} \sum_{k=1}^K \frac{x_{i_k}}{\sigma_{i_k}^2}. \quad (1)$$

with estimation MSE $\left(\sum_{k=1}^K 1/\sigma_{i_k}^2 \right)^{-1}$. But the BLUE scheme is impractical for wireless sensor networks because of the high communication and thus high energy cost, so quantization at the local sensors is essential. In this paper, we adopt a probabilistic quantization scheme [7] as well as a quasi-BLUE estimation scheme, based on which the optimal tradeoff between the number of active sensors and the energy allocated at each active sensor is addressed.

As shown in [7], let $\hat{x}(b)$ be an b -bit probabilistic quantization of bounded observation signal $x \in$

$[-W, W]$ with noise variance σ^2 , then $\hat{x}(b)$ is an unbiased estimator of θ with a variance

$$E \left(|\hat{x}(b) - \theta|^2 \right) \leq \sigma^2 + \frac{W^2}{(2^b - 1)^2} = \sigma^2 + \delta^2, \quad (2)$$

where $\delta^2 = W^2/(2^b - 1)^2$ for $b > 0$ denotes the upper bound of the quantization noise variance.

Now suppose all the observations of the K active sensors x_{i_k} are quantized into b_{i_k} -bits discrete messages $\hat{x}_{i_k}(b_{i_k})$, respectively, with the probabilistic quantization scheme. Then the quasi-BLUE estimator based on the quantized message has the following form,

$$\bar{\theta} = \left(\sum_{k=1}^K \frac{1}{\sigma_{i_k}^2 + \delta_{i_k}^2} \right)^{-1} \sum_{k=1}^K \frac{\hat{x}_{i_k}}{\sigma_{i_k}^2 + \delta_{i_k}^2}, \quad (3)$$

And the estimation MSE bound of the quasi-BLUE estimator is $\left(\sum_{k=1}^K 1/(\sigma_{i_k}^2 + \delta_{i_k}^2) \right)^{-1}$.

Assume that the channel between each sensor and the fusion center is corrupted with additive white Gaussian noise whose double-sided power spectrum density is given by $N_0/2$. In addition, the channel between sensor i_k and the fusion center experiences a pathloss proportional to $a_{i_k} = d_{i_k}^\alpha$, where d_{i_k} is the transmission distance and α is the passloss exponent. We further assume that sensors follow a time division multiple access scheme to send data to the fusion center. If sensor i_k sends b_{i_k} bits with quadrature amplitude modulation (QAM) with constellation size $2^{b_{i_k}}$ at a bit error probability $p_b^{i_k}$, then the total amount of required transmission energy [10] is

$$P_{i_k} = c_{i_k} a_{i_k} (2^{b_{i_k}} - 1), \quad (4)$$

with $c_{i_k} = 2N_f N_0 G_d \ln(2/p_b^{i_k})$, where N_f is the receiver noise figure, N_0 is the single-sided thermal noise spectral density, and G_d is a constant defined same as in [10].

Assuming all sensors have the same transmitter characteristic and the same desired negligible channel bit error probability, i.e., $c_{i_k} = c$ in Eq. (4) for all sensors, then $P_{i_k} = ca_{i_k} (2^{b_{i_k}} - 1)$. With the probabilistic quantization scheme and the quasi-BLUE fusion rule, our primary goal is to minimize the bound of the estimation MSE under the energy constraint, i.e.,

$$\begin{aligned} \min & \quad \left(\sum_{k=1}^K \frac{1}{\sigma_{i_k}^2 + \frac{c^2 a_{i_k}^2 W^2}{P_{i_k}^2}} \right)^{-1}, \\ \text{s.t.} & \quad \sum_{k=1}^K P_{i_k} \leq P_c, \quad P_{i_k} > 0, \quad k = 1, \dots, K, \end{aligned} \quad (5)$$

where P_c is the total energy allowed to use for all sensors, K is the number of active sensors and P_{i_k} is the allocated energy for the active sensor i_k .

III. EQUIVALENT UNIT-ENERGY MSE FUNCTION

As shown in Section II, the transmitted message from a sensor with observation noise variance σ^2 , transmission pathloss a , and transmission energy P is an unbiased estimation of the estimated parameter θ with the estimation MSE $D \leq \sigma^2 + \frac{c^2 a^2 W^2}{P^2}$. We denote the estimation MSE bound as

$$f(\sigma^2, a, P) = \sigma^2 + \frac{c^2 a^2 W^2}{P^2}. \quad (6)$$

Definition 1 (Equivalent Unit-Energy MSE function)

For a sensor with observation noise variance σ^2 , transmission pathloss a , and transmission energy P , the equivalent unit-energy MSE function is defined as

$$g(\sigma^2, a, P) \equiv P \cdot f(\sigma^2, a, P) = P \left(\sigma^2 + \frac{c^2 a^2 W^2}{P^2} \right). \quad (7)$$

Since the unit-energy MSE function $g(\sigma^2, a, P)$ is a convex function over P as shown in Proposition 1 below, we further define the optimal unit-energy MSE function $g^{opt}(\sigma^2, a)$ and the corresponding optimal transmission energy $P^{opt}(\sigma^2, a)$ for a sensor with observation noise variance σ^2 and transmission pathloss a as follows:

$$\begin{aligned} P^{opt}(\sigma^2, a) &= \arg \min_{P \in \mathbb{R}^+} g(\sigma^2, a, P) \\ g^{opt}(\sigma^2, a) &= \min_{P \in \mathbb{R}^+} g(\sigma^2, a, P) = g(\sigma^2, a, P^{opt}(\sigma^2, a)). \end{aligned} \quad (8)$$

Proposition 1 *The unit-energy MSE function $g(\sigma^2, a, P)$, the optimal unit-energy MSE function $g^{opt}(\sigma^2, a)$, the optimal transmission energy function $P^{opt}(\sigma^2, a)$ and the MSE function $f(\sigma^2, a, P)$ defined above have the following properties:*

- 1) $g(\sigma^2, a, P)$ is convex over P ($P > 0$).
- 2) $g^{opt}(\sigma^2, a)$ is achieved when the optimal transmission energy $P^{opt}(\sigma^2, a)$ is allocated, where

$$\begin{aligned} P^{opt}(\sigma^2, a) &= \frac{caW}{\sigma}, \\ g^{opt}(\sigma^2, a) &= 2ca\sigma W. \end{aligned} \quad (9)$$

- 3) The estimation MSE $f(\sigma^2, a, P^{opt}(\sigma^2, a))$ with the transmission energy $P^{opt}(\sigma^2, a)$ is,

$$f(\sigma^2, a, P^{opt}(\sigma^2, a)) = 2\sigma^2. \quad (10)$$

Proof:

- 1) The convexity of $g(\sigma^2, a, P)$ over P can be easily proved by checking $\partial^2 g(\sigma^2, a, P) / \partial^2 P > 0$.
- 2) Since $g(\sigma^2, a, P)$ is convex over P , it is minimized when $\partial g(\sigma^2, a, P) / \partial P = 0$, then we can get

$$\begin{aligned} P^{opt}(\sigma^2, a) &= \frac{caW}{\sigma}, \\ g^{opt}(\sigma^2, a) &= g(\sigma^2, a, P^{opt}(\sigma^2, a)) = 2ca\sigma W. \end{aligned}$$

- 3) According to the definition of $f(\sigma^2, a, P)$,

$$f(\sigma^2, a, P^{opt}(\sigma^2, a)) = 2\sigma^2. \quad \blacksquare$$

Denote the optimal quantization bit rate corresponding to the optimal transmission energy $P^{opt}(\sigma^2, a, c)$ as $b^{opt}(\sigma^2, a, c)$, then by Eq. (4) and (9),

$$b^{opt}(\sigma^2, a, c) = \log_2 \left(1 + \frac{W}{\sigma} \right). \quad (11)$$

IV. DISTRIBUTED ESTIMATION IN HOMOGENEOUS SENSOR NETWORKS

In homogeneous sensor networks, the noise variances for all sensors are identical, that is $\sigma_k^2 = \sigma^2$ ($k = 1, \dots, N$). And we assume the distances from all sensors to the fusion center are the same, thus the transmission pathloss are the same for all sensors, i.e., $a_k = a$. For this homogeneous sensor network model, it is easily verified that all active sensors should be allocated equal amount of energy $P_k = P$ to minimize the estimation MSE, so the number of active sensors is P_c/P , and the estimation MSE function is simplified to

$$E(\bar{\theta} - \theta)^2 \leq \left(\sum_{k=1}^K \frac{1}{\sigma^2 + \frac{c^2 a^2 W^2}{P^2}} \right)^{-1} = \frac{P \left(\sigma^2 + \frac{c^2 a^2 W^2}{P^2} \right)}{P_c}, \quad (12)$$

It is noticed that the numerator of the optimized target function in Eq. (12) is just the equivalent unit-energy MSE function $g(\sigma^2, a, P)$ defined in Section III. Hence, for homogeneous sensor networks, the optimal distributed estimation under the total energy constraint P_c can be solved using the concept of the equivalent unit-energy MSE function as follows:

- 1) For each sensor, the optimal transmission energy P^{opt} is identical and obtained by minimizing the corresponding equivalent unit-energy MSE function, as shown in Proposition 1,

$$P^{opt} = \frac{caW}{\sigma}. \quad (13)$$

- 2) The total number of active sensors K^{opt} under the total energy constraint P_c is

$$K^{opt} = \left\lfloor \frac{P_c}{P^{opt}} \right\rfloor. \quad (14)$$

It is obvious that the proposed method based on the equivalent unit-energy MSE function is optimal if P_c/P^{opt} is an integer, otherwise, it is quasi-optimal. Also, the proposed method is almost fully distributed.

V. DISTRIBUTED ESTIMATION IN HETEROGENEOUS SENSOR NETWORKS

In heterogeneous sensor networks, the observation noise variance of each sensor is σ_k^2 ($k = 1, \dots, N$), and the distance from sensor k to the fusion center is d_k . This scenario leads to the general problem stated in Eq. (5). Unfortunately, it can be easily verified that the optimal solution cannot be found in the closed form. Instead, we propose a quasi-optimal method to solve this problem, which is also based on the equivalent unit-energy MSE function. The procedure is stated as follows:

- 1) For each sensor $k \in [1, N]$, determine its optimal transmission energy P_k^{opt} and optimal unit-energy MSE function g_k^{opt} as shown in Proposition 1,

$$\begin{aligned} P_k^{opt} &= \frac{ca_k W}{\sigma_k}, \\ g_k^{opt} &= 2ca_k \sigma_k W. \end{aligned} \quad (15)$$

- 2) Let S_k ($k \in [1, N]$) denote the subset of all the sensors consisting of the first k sensors with the minimum optimal unit-energy MSE function,

$$\begin{cases} S_1 \subset S_2 \subset \dots \subset S_N, \\ g_i^{opt} \leq g_j^{opt}, \text{ if } i \in S_k \text{ and } j \in S_k^c, \end{cases} \quad (16)$$

where S_k^c denotes the complementary subset of S_k . Then the optimal number of active sensors K^{opt} under the total energy constraint P_c is determined by

$$K^{opt} = \max k \quad \text{s.t.} \quad \sum_{i \in S_k} P_i^{opt} \leq P_c. \quad (17)$$

In short, the whole solution is: the first K^{opt} sensors with the smallest unit-energy MSE function are chosen to quantize and transmit their observations, and the transmission energy of each chosen sensor is P_k^{opt} ($k \in S_{K^{opt}}$). Next, we will analyze the estimation MSE bound of the proposed method, which is stated in the following theorem. To simplify the statement, we assume $\sum_{k \in S_{K^{opt}}} P_k^{opt} = P_c$ in the subsequent analysis.

Theorem 1 *The estimation MSE of the proposed method based on the equivalent unit-energy MSE function under the total energy constraint P_c is*

$$E(\bar{\theta}_p - \theta)^2 \leq 2 \left(\sum_{k \in S_{K^{opt}}} \frac{1}{\sigma_k^2} \right)^{-1}, \quad (18)$$

where $\bar{\theta}_p$ denotes the estimation of the parameter θ by the proposed method, and $S_{K^{opt}}$ is obtained in Eq. (17).

Proof: By Proposition 1, we have

$$\begin{aligned} E(\bar{\theta}_p - \theta)^2 &\leq \left(\sum_{k \in S_{K^{opt}}} \frac{1}{f(\sigma_k^2, a_k, P^{opt}(\sigma_k^2, a_k))} \right)^{-1} \\ &= \left(\sum_{k \in S_{K^{opt}}} \frac{1}{2\sigma_k^2} \right)^{-1} = 2 \left(\sum_{k \in S_{K^{opt}}} \frac{1}{\sigma_k^2} \right)^{-1} \end{aligned} \quad (19)$$

This theorem gives the upper bound of the estimation MSE of the proposed method. It is shown that the proposed method is quasi-optimal (up to a factor of 2) when compared with the BLUE estimator using the same subset of active sensors without energy constraint.

As shown above, the performance bound of the proposed algorithm is analyzed, where the total energy is allocated to a special subset of sensors. Nevertheless, the remaining question is what performance can be achieved if the total energy P_c are allocated to any subset of the sensors. To answer this question, Theorem 2 states the lower bound of the estimation MSE by any quasi-BLUE estimation system with any subset of sensors under the total energy constraint P_c .

Theorem 2 *Assume any subset of sensors $S = \{i_1, \dots, i_k, \dots, i_{|S|}\}$ are used, where $i_k \in [1, N]$ and $|S|$ denotes the cardinality of the set S , i.e., the total number of sensors in the set S . The energy allocated to each active sensor $k \in S$ is P_k , such that $\sum_{k \in S} P_k = P_c$. Then the lower bound of the estimation MSE is*

$$E(\bar{\theta}_c - \theta)^2 > \left(\sum_{k \in S_{K^{opt}}} \frac{1}{\sigma_k^2} \right)^{-1}, \quad (20)$$

where $\bar{\theta}_c$ denotes the estimation of the parameter θ by the given subset of active sensors S under the given total energy constraint P_c , and $S_{K^{opt}}$ is the optimal subset of active sensors, obtained by our proposed algorithm as shown in Eq. (17) such that $\sum_{k \in S_{K^{opt}}} P_k^{opt} = P_c$.

Proof: Refer to Appendix A for the complete proof. ■

In conclusion, Theorem 1 shows that the upper bound of the estimation MSE of our proposed method is $E(\bar{\theta}_p - \theta)^2 \leq 2 \left(\sum_{k \in S_{K^{opt}}} 1/\sigma_k^2 \right)^{-1}$, and Theorem 2 shows that $\left(\sum_{k \in S_{K^{opt}}} 1/\sigma_k^2 \right)^{-1}$ is the lower bound of the estimation MSE of any quasi-BLUE estimator under the total energy constraint P_c , regardless of the subset of active sensors and the energy allocation among the active sensors. Therefore, the proposed algorithm gives a quasi-optimal tradeoff between the number of active sensors and the energy allocation of each active sensor, and its estimation MSE is within a factor 2 of the theoretical non-achievable lower bound.

VI. SIMULATION RESULTS

In this section, we will present some simulation results for the proposed algorithms in Section IV and V.

A. Homogeneous Sensor Networks

In this section, we simulate a homogeneous sensor network with $N = 500$ sensors, where the noise variances of all sensors are the same and the distances from all sensors to the fusion center are also the same. Without loss of generality, we assume the range of the observation signal is $[-1, 1]$, i.e., $W = 1$, and the distance is $d = 1$. Define the signal to noise ratio (SNR) as $SNR = 10 \log_{10}(W^2/\sigma^2)$ and generate different SNR by changing the observation noise variance σ^2 . Define the normalized energy as $P' = P/c = a(2^b - 1)$, where a and b are defined in Eq. (4). Assuming the total normalized energy constraint is $P'_c = 500$, Fig. 2 shows the estimation MSE with different quantization bit rates for the active sensors under different SNR, where different quantization bit rates represent different energy allocation for each sensor. For example, for the case of $SNR = 20$ dB, we can see from Fig. 2 that totally 71 out of all 500 sensors are active with 3-bit quantization message per sensor will produce the minimum estimation MSE among all the possible energy allocation strategies. From the results shown in Fig. 2, we also can see that for low SNR cases, such as 0 dB, 1-bit quantization per sensor will lead to the minimum estimation MSE. On the contrary, for high SNR cases, multiple-bit quantization per sensor will significantly decrease the estimation MSE compared to only 1-bit quantization per sensor under the same total energy constraint.

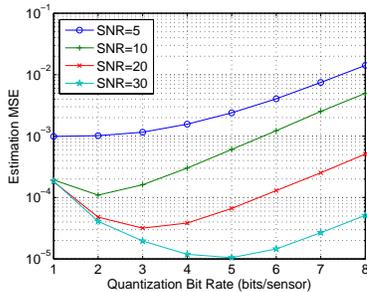


Fig. 2. The estimation MSE versus the quantization bit rate per sensor under different signal to noise ratio (SNR).

B. Heterogeneous Networks with Equal Distances

In this section, we simulate a heterogeneous sensor network with $N = 500$ sensors, where the noise variance of each sensor is different, which is assumed to be Chi-squared distribution with one degree of freedom, while

the distance from each sensor to the fusion center is the same. Same as before, we assume the range of the observation signal is $[-1, 1]$ and the distance from each sensor to the fusion center is $d = 1$. For any given total energy constraint, our proposed estimation method in Section V determines the subset of active sensors and the energy allocation for each active sensor to minimize the estimation MSE. In order to demonstrate the efficiency of the proposed method, we compare the proposed method with other two uniform schemes:

- 1) *Uniform-I*: For the given total energy constraint, the same subset of active sensors as that used by our proposed method is used, but all energy is uniformly allocated among all the active sensors.
- 2) *Uniform-II*: all sensors in the simulated network are used and all energy is uniformly allocated among all sensors.

Fig. 3 shows the estimation MSE by our proposed method, the *Uniform-I* method, the *Uniform-II* method, and the theoretical lower bound of the estimation MSE, under the total energy constraint. From Fig. 3, we can see that the proposed method outperforms the other two uniform schemes. Further, it also can be seen that the estimation MSE of our proposed method is within a factor 2 of the theoretical non-achievable lower bound.

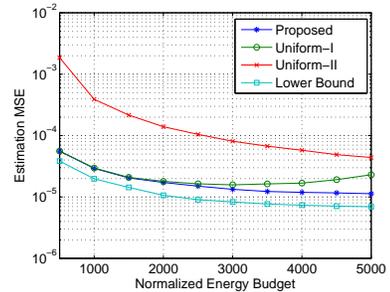


Fig. 3. The estimation MSE by the proposed method, *Uniform-I* method, *Uniform-II* method and the theoretical lower bound.

Note that both our proposed method and the *Uniform-I* method are based on the same subset of active sensors, and the only difference is that the optimal energy allocation is performed in our proposed method, while uniform energy allocation is performed in the *Uniform-I* method. Because of the heterogeneity of the network, a better estimation performance is obtained using our proposed method. To show how the heterogeneity of the sensor networks will influence the estimation performance, Fig. 4 plots the estimation MSE reduction of our proposed method compared with the *Uniform-I* method versus the normalized deviations of sensor noise variances, which is used as a measure of the heterogeneity of the sensor network and is defined as

$\alpha = \sqrt{\text{Var}(\sigma^2)}/E(\sigma^2)$. From Fig. 4, we conclude that when compared with the *Uniform-I* method, the amount of estimation MSE reduction of our proposed method becomes more significant when the local sensor noise variances become more heterogeneous.

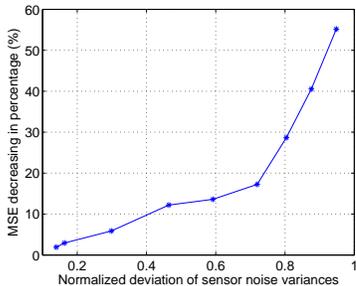


Fig. 4. The estimation MSE reduction in percentage of the proposed method compared with the *Uniform-I* method.

C. Heterogeneous Networks with Random Distances

In this part of the simulation, we relax the assumption in Section VI-B that the distance from each sensor to the fusion center is the same. We assume the distance d_k from the sensor k to the fusion center is identically, independently and uniformly distributed from 1 to 10, i.e., $d_k \sim U[1, 10]$. Fig. 5 shows the estimation MSE by our proposed method, the *Uniform-I* method, the *Uniform-II* method, and the theoretical lower bound of the estimation MSE, under the total energy constraint. From Fig. 5, again, we can see that the proposed method outperforms the other two uniform schemes, and that the estimation MSE of our proposed method is within a factor 2 of the theoretical non-achievable lower bound.

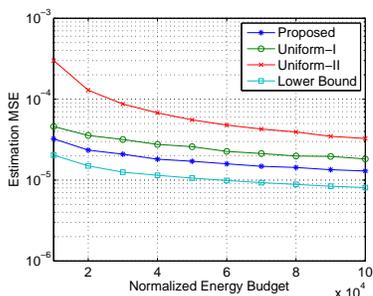


Fig. 5. The estimation MSE by the proposed method, *Uniform-I* method, *Uniform-II* method and the theoretical lower bound.

VII. CONCLUSIONS

In this paper, we considered distributed parameter estimation in energy-constrained wireless sensor networks. For a given constraint on the allowable energy to use

for all sensors, we studied the optimal tradeoff between the number of active sensors and the energy allocated to each active sensor to minimize the estimation MSE. To facilitate the solution, a concept of equivalent unit-energy MSE function was introduced. Then, an optimal distributed estimation algorithm for homogeneous sensor networks and a quasi-optimal distributed estimation algorithm for heterogeneous sensor networks, which are both based on the equivalent unit-energy MSE function, were proposed. Simulation results show that a significant reduction in estimation MSE is achieved by our proposed algorithm when compared with other uniform methods.

APPENDIX A

PROOF OF THEOREM 2

Assume a subset of sensors $S_a = \{i_1, \dots, i_k, \dots, i_{|S_a|}\}$ are used, where $i_k \in [1, N]$, and the energy allocated to each active sensor $k \in S_a$ is P_k^a , and $\sum_{k \in S_a} P_k^a = P_c$. Denote this estimation system as C_a , the estimation of θ as $\bar{\theta}_a$, and its estimation MSE as D_a , so the objective is to show that $D_a = E(\bar{\theta}_a - \theta)^2 > \left(\sum_{k \in S_{K^{opt}}} 1/\sigma_k^2\right)^{-1}$, where $S_{K^{opt}}$ is the optimal subset of active sensors, obtained by our proposed algorithm as shown in Eq. (17) such that $\sum_{k \in S_{K^{opt}}} P_k^{opt} = P_c$. The basic idea to prove this statement is to construct another quasi-BLUE estimation system, denoted as C_b , with estimation MSE D_b such that $D_a \geq D_b > \left(\sum_{k \in S_{K^{opt}}} 1/\sigma_k^2\right)^{-1}$. The estimation system C_b is constructed as follows: only the subset of sensors $S_b = S_{K^{opt}}$ are used and the energy allocated to each sensor $k \in S_b$ is P_k^b , and

$$P_k^b = \begin{cases} \max(P_k^a, P_k^{opt}), & \text{if } k \in S_{K^{opt}} \cap S_a, \\ P_k^{opt}, & \text{if } k \in S_{K^{opt}} \setminus S_a, \\ 0, & \text{otherwise,} \end{cases} \quad (21)$$

where, $k \in S_{K^{opt}} \setminus S_a$ means that $k \in S_{K^{opt}}$ but $k \notin S_a$.

(1) Show that $D_b > \left(\sum_{k \in S_{K^{opt}}} 1/\sigma_k^2\right)^{-1}$.

Since in the constructed system C_b , only the sensors in the subset $S_{K^{opt}}$ are active and limited energy P_k^b are used for each sensor $k \in S_{K^{opt}}$, and $D_0 = \left(\sum_{k \in S_{K^{opt}}} 1/\sigma_k^2\right)^{-1}$ is the lower bound of the estimation MSE of BLUE estimator using the subset of sensors $S_{K^{opt}}$, so $D_b > D_0 = \left(\sum_{k \in S_{K^{opt}}} 1/\sigma_k^2\right)^{-1}$.

(2) Show that $D_b \leq D_a$.

Divide S_a into three disjoint subset S_{a1} , S_{a2} and S_{a3} as follows:

$$\begin{aligned} S_{a1} &= \{k : P_k^a \geq P_k^{opt}, \text{ and } k \in S_{K^{opt}} \cap S_a\}, \\ S_{a2} &= \{k : P_k^a < P_k^{opt}, \text{ and } k \in S_{K^{opt}} \cap S_a\}, \\ S_{a3} &= S_a \setminus S_{K^{opt}}. \end{aligned} \quad (22)$$

Similarly, divide S_b into three disjoint subset S_{b1} , S_{b2} and S_{b3} as follows:

$$\begin{aligned} S_{b1} &= \{k : P_k^a \geq P_k^{opt}, \text{ and } k \in S_{K^{opt}} \cap S_a\}, \\ S_{b2} &= \{k : P_k^a < P_k^{opt}, \text{ and } k \in S_{K^{opt}} \cap S_a\}, \\ S_{b3} &= S_{K^{opt}} \setminus S_a. \end{aligned} \quad (23)$$

Proposition 2 According to the definitions of $S_{K^{opt}}$, S_a , S_b and P_k^b in Eq. (17), (21), (22), (23), it is easy to see that:

- 1) $S_{a1} \cup S_{a2} \cup S_{a3} = S_a$ and $S_{b1} \cup S_{b2} \cup S_{b3} = S_b$,
- 2) $S_{b1} = S_{a1}$ and $P_k^b = P_k^a \geq P_k^{opt}$ for any $k \in S_{b1}$,
- 3) $S_{b2} = S_{a2}$ and $P_k^b = P_k^{opt} > P_k^a$ for any $k \in S_{b2}$,
- 4) $S_b = S_{K^{opt}}$ and $S_{a3} \subseteq S_{K^{opt}}$, thus for any $i \in S_b$ and $j \in S_{a3}$, $g(\sigma_i^2, a_i, P_i^b) \leq g(\sigma_j^2, a_j, P_j^a)$ according to Eq. (16). Let $g_1 = \max_{i \in S_b} g(\sigma_i^2, a_i, P_i^b)$ and $g_2 = \min_{j \in S_{a3}} g(\sigma_j^2, a_j, P_j^a)$, then $g_1 \leq g_2$.

Let $D_a = 1/D'_a$, and $D_b = 1/D'_b$. Expressing D'_a and D'_b with the concept of the equivalent unit-energy MSE functions as follows:

$$\begin{aligned} D'_a &= \sum_{k \in S_{a1} \cup S_{a2} \cup S_{a3}} \frac{P_k^a}{g(\sigma_k^2, a_k, P_k^a)} \\ D'_b &= \sum_{k \in S_{b1} \cup S_{b2} \cup S_{b3}} \frac{P_k^b}{g(\sigma_k^2, a_k, P_k^b)} \end{aligned} \quad (24)$$

according to Proposition 2, then

$$\begin{aligned} D'_b - D'_a &= \left(\sum_{k \in S_{b1}} \frac{P_k^b}{g(\sigma_k^2, a_k, P_k^b)} - \sum_{k \in S_{a1}} \frac{P_k^a}{g(\sigma_k^2, a_k, P_k^a)} \right) \\ &+ \left(\sum_{k \in S_{b2}} \frac{P_k^b}{g(\sigma_k^2, a_k, P_k^b)} - \sum_{k \in S_{a2}} \frac{P_k^a}{g(\sigma_k^2, a_k, P_k^a)} \right) \\ &+ \sum_{k \in S_{b3}} \frac{P_k^b}{g(\sigma_k^2, a_k, P_k^b)} - \sum_{k \in S_{a3}} \frac{P_k^a}{g(\sigma_k^2, a_k, P_k^a)} \\ &= \left(\sum_{k \in S_{b2}} \frac{P_k^a}{g(\sigma_k^2, a_k, P_k^{opt})} - \sum_{k \in S_{a2}} \frac{P_k^a}{g(\sigma_k^2, a_k, P_k^a)} \right) \\ &+ \sum_{k \in S_{b2}} \frac{P_k^{opt} - P_k^a}{g(\sigma_k^2, a_k, P_k^{opt})} + \sum_{k \in S_{b3}} \frac{P_k^{opt}}{g(\sigma_k^2, a_k, P_k^{opt})} \\ &- \sum_{k \in S_{a3}} \frac{P_k^a}{g(\sigma_k^2, a_k, P_k^a)} \\ &\geq \sum_{k \in S_{b2}} \frac{P_k^{opt} - P_k^a}{g(\sigma_k^2, a_k, P_k^{opt})} + \sum_{k \in S_{b3}} \frac{P_k^{opt}}{g(\sigma_k^2, a_k, P_k^{opt})} \\ &- \sum_{k \in S_{a3}} \frac{P_k^a}{g(\sigma_k^2, a_k, P_k^a)}, \\ &\geq \left(\sum_{k \in S_{b2}} (P_k^{opt} - P_k^a) + \sum_{k \in S_{b3}} P_k^{opt} \right) \frac{1}{g_1} \\ &- \left(\sum_{k \in S_{a3}} P_k^a \right) \frac{1}{g_2}, \\ &\geq \left(\sum_{k \in S_{b2} \cup S_{b3}} P_k^{opt} \right) \frac{1}{g_1} - \left(\sum_{k \in S_{a2} \cup S_{a3}} P_k^a \right) \frac{1}{g_2}. \end{aligned} \quad (25)$$

From the total energy constraint, we have

$$\sum_{k \in S_{a1} \cup S_{a2} \cup S_{a3}} P_k^a = \sum_{k \in S_{b1} \cup S_{b2} \cup S_{b3}} P_k^{opt} = P_c \quad (26)$$

Since $S_{b1} = S_{a1}$ and $P_k^a \geq P_k^{opt}$ for any $k \in S_{a1}$ as shown in Eq. (22) and Proposition 2, then

$$\sum_{k \in S_{a2} \cup S_{a3}} P_k^a \leq \sum_{k \in S_{b2} \cup S_{b3}} P_k^{opt}, \quad (27)$$

thus,

$$\begin{aligned} D'_b - D'_a &\geq \left(\sum_{k \in S_{b2} \cup S_{b3}} P_k^{opt} \right) \frac{1}{g_1} - \left(\sum_{k \in S_{a2} \cup S_{a3}} P_k^a \right) \frac{1}{g_2} \\ &\geq 0, \end{aligned} \quad (28)$$

therefore,

$$D_a \geq D_b. \quad (29)$$

From (1) and (2) above, we get

$$D_a \geq D_b > \left(\sum_{k \in S_{K^{opt}}} \frac{1}{\sigma_k^2} \right)^{-1}. \quad (30)$$

REFERENCES

- [1] S. Kumar, F. Zhao, and D. Shepherd, "Special issue on collaborative information processing in microsensor networks," *IEEE Signal Processing Magazine*, vol. 19, no. 2, pp. 13–14, Mar. 2002.
- [2] H. Papadopoulos, G. Wornell, and A. Oppenheim, "Sequential signal encoding from noisy measurements using quantizers with dynamic bias control," *IEEE Transactions on Information Theory*, vol. 47, no. 3, pp. 978–1002, Mar. 2001.
- [3] A. Ribeiro and G. Giannakis, "Bandwidth-constrained distributed estimation for wireless sensor networks, part i: Gaussian case," *IEEE Transactions on Signal Processing*, vol. 54, no. 3, pp. 1131–1143, Mar. 2006.
- [4] Z.-Q. Luo, "Universal decentralized estimation in a bandwidth constrained sensor network," *IEEE Transactions on Information Theory*, vol. 51, no. 6, pp. 2210–2219, Jun. 2005.
- [5] Z.-Q. Luo and J.-J. Xiao, "Decentralized estimation in an inhomogeneous sensing environment," *IEEE Transactions on Information Theory*, vol. 51, no. 10, pp. 3564–3575, Oct. 2005.
- [6] J. Li and G. AlRegib, "Rate-constrained distributed estimation in wireless sensor networks," *IEEE Transaction on Signal Processing*, vol. 55, no. 5, pp. 1634–1643, May 2007.
- [7] J.-J. Xiao, S. Cui, Z.-Q. Luo, and A. Goldsmith, "Power scheduling of universal decentralized estimation in sensor networks," *IEEE Transactions on Signal Processing*, vol. 54, no. 2, pp. 413–422, Feb. 2006.
- [8] A. Krasnoperov, J.-J. Xiao, and Z.-Q. Luo, "Minimum energy decentralized estimation in sensor network with correlated sensor noise," *EURASIP Journal on Wireless Communications and Networking*, vol. 5, no. 4, pp. 473–482, 2005.
- [9] S. Kay, *Fundamentals of Statistical Signal Processing, Volume 1: Estimation Theory*. Englewood Cliffs, New Jersey: Prentice Hall, 1993.
- [10] S. Cui, A. J. Goldsmith, and A. Bahai, "Energy-constrained modulation optimization," *IEEE Trans. Wireless Commun.*, vol. 4, no. 5, pp. 2349–2360, Sep. 2005.