Maximizing Network Lifetime for Estimation in Multi-Hop Wireless Sensor Networks

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Abstract— **In this paper, we consider distributed estimation in energy-limited wireless sensor networks from *lifetime-distortion* perspective, where the goal is to maximize the network lifetime for a given distortion requirement. To take into account both local quantization and multi-hop transmission, which are essential to save transmission energy and thus prolong the network lifetime, the network lifetime maximization problem is formulated as a nonlinear programming (NLP) problem, where there are three factors needed to be optimized jointly: (i) source coding at each sensor, (ii) source throughput of each sensor, and (iii) multi-hop routing path. Furthermore, we show that this NLP problem can be decoupled without loss of optimality and reformulated as a linear programming (LP) problem. The proposed algorithm is optimal and the simulation results show that a significant gain is achieved by the proposed algorithm compared with heuristic methods.**

I. Introduction

A common goal of most applications, such as environment monitoring, battlefield surveillance, and health care, in wireless sensor networks (WSN) is to reconstruct the underlying physical phenomenon, e.g., temperature, based on sensor measurements. Distributed estimation of unknown deterministic parameters by a set of distributed sensor nodes and a fusion center has become an important topic in signal processing research for wireless sensor networks [1], where sensor nodes collect real-valued data, perform a local data compression, and send the resulting messages to the fusion center, while the fusion center combines the received messages to produce a final estimation of the observed parameter.

Subject to the resource (bandwidth and energy) limitation nature of wireless sensor networks, several bandwidth-constrained distributed estimation algorithms [2]–[5] have been investigated recently. In [2], a class of maximum likelihood estimators (MLE) was proposed to attain a variance that is close to the clairvoyant estimator when the observations are quantized to one bit. Without the knowledge of noise distribution, the work of [3] and [4] proposed several universal (pdf-unaware) decentralized estimation systems based on best linear unbiased estimation (BLUE) rule. In [5], quasi-optimal distributed parameter estimation algorithms are proposed to minimize the estimation mean square error (MSE) under a total rate constraint. Also, the minimal-energy distributed estimation problem has been recently considered in [6]–[9]. In [6], [7], the total sensor transmission energy is minimized by selecting the optimal quantization levels while meeting the target estimation MSE requirements. The work of [8] provided a solution to minimal-energy distributed estimation by exploiting long-term noise variance statistics. by exploiting long-term noise variance statistics. The work of [9] proposed quasi-optimal distributed parameter estimation algorithms to minimize the estimation mean square error (MSE) under a given allowable energy budget for all sensors.

All the aforementioned algorithms address distributed estimation from either *rate-distortion* perspective or *energy-distortion* perspective. To the best of our knowledge, network lifetime issue for estimation application in wireless sensor networks has not yet been addressed explicitly in the literature. In this paper, we study the *lifetime-distortion* issue for estimation application in energy-limited sensor networks. In energy-limited wireless sensor networks, both local quantization and multi-hop transmission are essential to save transmission energy and thus prolong the network lifetime. To maximize the network lifetime for estimation application, three factors are needed to be optimized together: *(i) source coding, i.e., quantization level of each observation, (ii) source throughput, i.e. total number of observations or total information bits generated by each sensor, and (iii) multi-hop routing path to transmit the observations from all sensors to the fusion center*. This problem can be formulated as a nonlinear programming (NLP) problem. Fortunately, source coding optimization can be decoupled from source throughput and multi-hop routing optimization and solved by introducing a concept of equivalent 1-bit MSE function. Based on optimal source coding, the source throughput and multi-hop routing path optimization can be formulated as a linear programming (LP) problem, which is easy to solve.

The rest of the paper is organized as follows. Section II introduces the system model. Section III formulates the network lifetime maximization problem as a nonlinear programming (NLP) problem, and then decouples the original problem into two sub-problems without compromising the optimality. Then in Section IV and section V, we solve the two sub-problems, i.e., (i) source coding optimization, and (ii) joint source throughput and multi-hop routing optimization, respectively. Section VI gives some simulation results that demonstrate the efficiency of the proposed algorithms. Finally, conclusions are given in Section VII.

II. System Model

We consider a dense sensor network including N distributed sensor nodes and a fusion center, denoted as node $N + 1$, to

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observe and estimate an unknown parameter θ.

A. Estimation Model

First, each sensor k can make observations on the unknown parameter θ, which are corrupted by additive noise and described by

$$x_k = \theta + n_k, \quad k = 1, \cdots, N, \tag{1}$$

where the observation noises of all sensors n_k are assumed to be zero mean, spatially uncorrelated with variance σ_k^2, while the noise at each sensor is assumed to be temporally i.i.d distributed, otherwise unknown.

Subject to severe bandwidth and energy limitations, each sensor is prevented from transmitting real-valued (analogy) data to the fusion center, that is to say, a local quantization $m_k = Q_k(x_k)$ is performed before transmission, where $Q_k(x_k)$ is a quantization function. Assume there are K observations $(m_1, m_2, \cdots, m_K)$ available at the fusion center, fusion center produces a final estimation of θ by combining all the available observations using a fusion function $f : \bar{\theta} = f(m_1, m_2, \cdots, m_K)$.

Assume the observation signal is bounded, i.e., $x_k \in [-W, W]$, we adapt a probabilistic quantization scheme [6] at each sensor to make the local quantization, as well as a quasi-BLUE estimation scheme at the fusion center to make the final estimation. Suppose all the observations of K active sensors $x_k (k = 1, \cdots, K)$ are quantized into b_k-bits discrete messages $m_k(b_k)$ respectively with the probabilistic quantization scheme, then the variance of the quantized message $E(m_k(b_k) - \theta)^2 \leq \sigma_k^2 + \delta_k^2(b_k) := \pi_k^2(\sigma_k^2, b_k)$, where $\delta_k^2(b_k) = W^2/\left(2^{b_k} - 1\right)^2$ denotes the upper bound of the quantization noise variance. The quasi-BLUE estimator based on the quantized message has the following form:

$$\bar{\theta} = \left(\sum_{k=1}^{K} \frac{1}{\pi_k^2(\sigma_k^2, b_k)} \right)^{-1} \sum_{k=1}^{K} \frac{m_k}{\pi_k^2(\sigma_k^2, b_k)}. \tag{2}$$

Notice that $\bar{\theta}$ is an unbiased estimator of θ since every m_k is unbiased, and the estimation MSE of the quasi-BLUE estimator is

$$E(\bar{\theta} - \theta)^2 \leq \left(\sum_{k=1}^{K} \frac{1}{\pi_k^2(\sigma_k^2, b_k)} \right)^{-1}. \tag{3}$$

B. Energy Model

Assume sensor nodes can adjust their transmission power to control the transmission range. The energy consumed by sensor i to reliably transmit a b-bit message to sensor j is

$$e(b) = c \cdot b \cdot d_{i,j}^{\alpha}, \tag{4}$$

where c is a system constant denoting the energy required by transmitter amplifier to transmit 1-bit to one meter, α is the path loss exponent depending on the medium properties, and $d_{i,j}$ is the distance between sensor i and sensor j.

III. Network Lifetime for Estimation

Network lifetime is a critical concern in the design of wireless sensor networks. In this section, we first define the network lifetime and then formulate the network lifetime maximization problem.

A. Function-based Network Lifetime

In the literature, many lifetime definitions are used, such as, duration of time until the first sensor failure due to battery depletion, fraction of surviving nodes in a network, and mean expiration time etc. Instead, in this paper, we introduce a notion of function-based network lifetime, which focuses on whether the network can perform a given task instead of whether any individual sensor is dead.

Definition 1 (Function-based Network Lifetime) *For estimation application, the network is considered functional if it can produce an estimation satisfying a distortion requirement D_r, otherwise it is nonfunctional. The network lifetime L is defined as the estimation task cycles accomplished before the network becomes nonfunctional.*

At different estimation cycles, the parameter θ is assumed to be unrelated, and the estimation at each cycle is performed independently using only the observations made by all sensors in the given estimation cycle. Based on the system model in Section II, assume a sensor network with N sensors, each with observation noise variance σ_k^2 $(k = 1, \cdots, N)$. To satisfy the given estimation distortion requirement D_r, at each estimation cycle, a subset of the sensors is required to observe the parameter θ and transmit their quantized measurements to the fusion center to make the final estimation.

Proposition 1 *Assume sensor k $(k = 1, \cdots, N)$ can make a total of M_k measurements and quantize its measurements using probabilistic quantization scheme to $b_{k,i}$ $(i = 1, \cdots, M_k)$ bits, respectively, before it depletes. Then the function-based network lifetime L for estimation application is bounded as follows:*

$$L \leq D_r \left(\sum_{k=1}^{N} \sum_{i=1}^{M_k} \frac{1}{\pi_k^2(\sigma_k^2, b_{k,i})} \right), \tag{5}$$

where, N, σ_k^2, M_k, $b_{k,i}$, D_r and $\pi_k^2(\sigma_k^2, b_{k,i})$ are defined as before.

Proof: Assume the network lifetime for this network is L. At each estimation cycle $l \in [1, L]$, denote the subset of observations each sensor k makes and sends to the fusion center is $O_{k,l}$. Then for any sensor $k \in [1, N]$, we have

$$\begin{aligned} &O_{k,i} \cap O_{k,j} = \emptyset, \quad \forall i, j \in [1, L], \text{ and } i \neq j, \\ &\bigcup_{l=1}^{L} O_{k,l} \subseteq \{1, \cdots, M_k\}, \end{aligned} \tag{6}$$

and for any estimation cycle $l \in [1, L]$, we have

$$\left(\sum_{k=1}^{N} \sum_{i \in O_{k,l}} \frac{1}{\pi_k^2(\sigma_k^2, b_{k,i})} \right)^{-1} \leq D_r. \tag{7}$$

So,

$$\sum_{l=1}^{L}\sum_{k=1}^{N}\sum_{i\in O_{k,l}}\frac{1}{\pi_k^2(\sigma_k^2,b_{k,i})}\geq\frac{L}{D_r},\tag{8}$$

i.e.,

$$\sum_{k=1}^{N}\sum_{i=1}^{M_k}\frac{1}{\pi_k^2(\sigma_k^2,b_{k,i})}\geq\frac{L}{D_r},\tag{9}$$

therefore,

$$L\leq D_r\left(\sum_{k=1}^{N}\sum_{i=1}^{M_k}\frac{1}{\pi_k^2(\sigma_k^2,b_{k,i})}\right).\tag{10}$$

■

Based on the system model and the definition of function-based network lifetime, the objective of this paper is to maximize the function-based network lifetime bound shown in Eq. (5) under the energy resource constraint of each sensor.

B. Nonlinear programming (NLP) Formulation

Model the wireless sensor network as a directed graph $G(V,E)$, where V is the set consisting of all the N sensor nodes and the fusion center (node $N+1$), i.e., $V=[1,N+1]$, E is the set of directed links in the network. An edge $(i,j)\in E$ iff $d_{i,j}\leq R$, where $d_{i,j}$ is the distance between node i and node j, and R is the maximum transmission range. The link cost, denoted as $C_{i,j}$, to transmit a unit bit information from node i to node j depends on the distance $d_{i,j}$ between them based on the energy model used in Eq. (4) as follows:

$$C_{i,j}=\begin{cases}cd_{i,j}^{\alpha}, & \text{if } d_{i,j}\leq R\\ +\infty, & \text{otherwise}\end{cases}\tag{11}$$

Assume each sensor has a limited energy supply P_k $(k=1,\cdots,N)$, according to network lifetime bound shwon in Eq. (5), the network lifetime maximization problem can be formulated as a nonlinear programming (NLP) problem as follows:

$$\text{maximize}\quad D_r\left(\sum_{k=1}^{N}\sum_{i=1}^{M_k}\frac{1}{\pi_k^2(\sigma_k^2,b_{k,i})}\right)\tag{12}$$

subject to

$$\sum_{i=1,i\neq k}^{N}f_{i,k}+S_k=\sum_{j=1,j\neq k}^{N+1}f_{k,j},\quad\forall k\in[1,N]\tag{13}$$

$$\sum_{j=1,j\neq k}^{N+1}f_{k,j}C_{k,j}\leqslant P_k,\quad\forall k\in[1,N]\tag{14}$$

$$S_k=\sum_{i=1}^{M_k}b_{k,i},\quad\forall k\in[1,N]\tag{15}$$

where

$$\begin{array}{ll}S_k\geq 0, M_k\geq 0, & \forall k\in[1,N]\\ b_{k,i}\geq 0, & \forall k\in[1,N], i\in[1,M_k]\\ f_{i,j}\geq 0, & \forall i\in[1,N], j\in[1,N+1]\end{array}\tag{16}$$

S_k denotes the source throughput of sensor node k, i.e., the total amount of data generated at sensor node k. $f_{i,j}$ denotes the amount of data transmitted from sensor node i to sensor node j. Eq. (13) and Eq. (14) represent two constraints of the optimization problem: (i) flow conservation, i.e., the amount of data transmitted by a sensor node is equal to the sum of the amount of data received by the sensor node and the amount of data generated by the sensor node itself, and (ii) energy constraint, i.e., the amount of data transmitted by a sensor node is limited by the energy supply of the sensor node.

C. Separation of Source Coding with Routing

To maximize the objective function in Eq. (12), there are three factors needed to be optimized together: (i) source coding at each sensor, i.e., quantization level $b_{k,i}$ for each observation i of each sensor k, (ii) source throughput S_k of each sensor k, and (iii) multi-hop routing path, i.e., the feasible network flow f. Fortunately, it is easy to see that given source throughput S_k, the source coding optimization can be decoupled from the multi-hop routing optimization since the objective function in Eq. (12) only depends on the source throughput and source coding, but does not depend on how the source data is transmitted to the fusion center. Therefore, we can optimize the original problem stated in Eq. (12, 13, 14, 15) in two steps without loss of optimality: (i) optimizing the source coding for given source throughput, and (ii) optimizing the source throughput and multi-hop routing path jointly, based on the optimal source coding.

IV. Source Coding Optimization

In this section, we optimize the source coding for a given source throughput S_k of each sensor $k\in[1,N]$. Mathematically, the problem is formulated as follows:

$$\begin{array}{ll}\max & D_r\left(\sum_{k=1}^{N}\sum_{i=1}^{M_k}\frac{1}{\pi_k^2(\sigma_k^2,b_{k,i})}\right)\\ s.t. & \sum_{i=1}^{M_k}b_{k,i}=S_k,\quad\forall k\in[1,N],\end{array}\tag{17}$$

where, $M_k\geq 0$ and $b_{k,i}\geq 0$ defined as before are variables to be optimized.

A. Equivalent 1-bit MSE Function

To facilitate the solution to Eq. (17), we first introduce a concept of equivalent 1-bit MSE function.

Definition 2 (Equivalent 1-bit MSE Function) *For a quantized message from a sensor with observation noise variance* σ^2 *and quantization bit rate* b*, the estimation variance bound is* $\pi^2(\sigma^2,b):=\sigma^2+\frac{W^2}{(2^b-1)^2}$ *as shown in Section II-A. Then, the equivalent 1-bit MSE function is defined as*

$$g(\sigma^2,b):=b\cdot\pi^2(\sigma^2,b)=b\cdot\left(\sigma^2+\frac{W^2}{(2^b-1)^2}\right).\tag{18}$$

With this definition, a b-bit quantization sensor with estimation MSE $\pi^2(\sigma^2,b)$ is equivalent to b equivalent 1-bit sensors, each with the same estimation MSE $g(\sigma^2,b)$. That is why $g(\sigma^2,b)$ is called equivalent 1-bit MSE function.

It is easy to show that $g(\sigma^2,b)$ is convex over $b>0$. Then, we further define the optimal 1-bit MSE function $g^{opt}(\sigma^2)$

and the corresponding optimal quantization bit rate $b^{opt}(\sigma^2)$ as follows:

$$\begin{aligned} b^{opt}(\sigma^2) &= \arg\min_{b\in\mathbb{Z}^+} g(\sigma^2, b), \\ g^{opt}(\sigma^2) &= \min_{b\in\mathbb{Z}^+} g(\sigma^2, b) = g(\sigma^2, b^{opt}(\sigma^2)), \end{aligned} \tag{19}$$

where the minimization involves just a simple one-dimensional numerical search. Note that $b \in \mathbb{Z}^+$ in Eq. (19) since the quantization bit rate must be integer in practice.

B. Upper Bound of Network Lifetime

Based on the definitions above, the network lifetime bound for estimation can be reformulated as a linear function of the source throughput S_k ($k = 1, \cdots, N$) as shown in Theorem 1 below.

Theorem 1 *Given the source throughput S_k of all sensor nodes $k \in [1, N]$ and the estimation distortion requirement D_r, the bound of function-based network lifetime for estimation is*

$$L \le D_r \left(\sum_{k=1}^{N} \frac{S_k}{g_k^{opt}(\sigma_k^2)} \right), \tag{20}$$

where, $g_k^{opt}(\sigma_k^2)$ is the optimal 1-bit MSE function of sensor node k.

Proof: Assume sensor $k \in [1, N]$ makes a total of M_k measurements, each with quantization bit rate $b_{k,i}$, respectively, such that $\sum_{i=1}^{M_k} b_{k,i} \le S_k$. Then as shown in Eq. (5), the network lifetime bound is

$$L \le D_r \left(\sum_{k=1}^{N} \sum_{i=1}^{M_k} \frac{1}{\pi_k^2(\sigma_k^2, b_{k,i})} \right). \tag{21}$$

According to the definition of $g(\sigma^2, b)$ and $g^{opt}(\sigma^2)$ in Eq. (18, 19) and the source throughput constraints in Eq. (17),

$$\begin{aligned} L &\le D_r \left(\sum_{k=1}^{N} \sum_{i=1}^{M_k} \frac{1}{\pi_k^2(\sigma_k^2, b_{k,i})} \right) \\ &= D_r \left(\sum_{k=1}^{N} \sum_{i=1}^{M_k} \frac{b_{k,i}}{g_k(\sigma_k^2, b_{k,i})} \right) \\ &\le D_r \left(\sum_{k=1}^{N} \sum_{i=1}^{M_k} \frac{b_{k,i}}{g_k^{opt}(\sigma_k^2)} \right) \\ &\le D_r \left(\sum_{k=1}^{N} \frac{S_k}{g_k^{opt}(\sigma_k^2)} \right) \end{aligned} \tag{22}$$

thus, the theorem is proved. ■

Note that the equality in Eq. (22) is achieved when each sensor node adopts optimal source coding, i.e., optimal quantization bit rate $b^{opt}(\sigma^2)$ to quantize its observations. As shown before, the optimal quantization bit rate $b^{opt}(\sigma^2)$ of each sensor can be easily obtained by minimizing its equivalent 1-bit MSE function, which only depends on its own observation noise variance, therefore, this optimization can be done in a completely distributed manner.

V. Joint Optimization of Source Throughput and Multi-Hop Routing

As shown in Eq. (20) in Theorem 1, the network lifetime bound depends on the source throughput S_k for all sensors $k \in [1, N]$, which are unknown variables to be optimized. In multi-hop wireless sensor networks, each sensor not only transmits the data generated by itself, but also relays the data for other sensors. Since the total amount of data each sensor can transmit and relay is limited by the energy supply of the sensor node, the source throughput of each sensor and the multi-hop routing need to be optimized jointly.

A. Linear Programming (LP) Formulation

As shown in Theorem 1, the nonlinear objective function in Eq. (12) can be reformulated as a linear function of the source throughput S_k ($k \in [1, N]$) by the optimal source coding, then, the original network lifetime bound maximization problem shown in Section III-B can be reformulated as a linear programming (LP) problem as follows:

$$\text{maximize} \quad D_r \left(\sum_{k=1}^{N} \frac{S_k}{g_k^{opt}(\sigma_k^2)} \right) \tag{23}$$

subject to

$$\sum_{i=1, i\neq k}^{N} f_{i,k} + S_k = \sum_{j=1, j\neq k}^{N+1} f_{k,j}, \quad \forall k \in [1, N] \tag{24}$$

$$\sum_{j=1, j\neq k}^{N+1} f_{k,j} C_{k,j} \leqslant P_k, \quad \forall k \in [1, N] \tag{25}$$

where

$$\begin{aligned} &S_k \ge 0, && \forall k \in [1, N] \\ &f_{i,j} \ge 0, && \forall i \in [1, N], j \in [1, N+1] \end{aligned} \tag{26}$$

and all variables are defined as before.

In summary, the network lifetime bound maximization for estimation can be formulated as a linear programming problem as shown in Eq. (23, 24, 25), which can be easily solved using any LP solver, such as [10] used in our simulations.

B. Special Case: Homogeneous Networks

In homogeneous wireless sensor networks, where each sensor has the same observation noise variance, i.e., $\sigma_k^2 = \sigma^2$ ($k = 1, \cdots, N$), single-hop routing path, i.e., all sensors transmit their observations to the fusion center directly, can maximize the network lifetime bound as shown in Proposition 2 below.

Proposition 2 *In a homogeneous wireless sensor network with N sensor nodes and observation noise variance σ^2, the optimal routing solution to network lifetime maximization problem shown in Eq. (23, 24, 25) is single-hop routing. Further more, assume sensor k ($k = 1, \cdots, N$) has P_k amount of energy supply, then the network lifetime bound for estimation with distortion requirement D_r is*

$$L \le D_r \left(\sum_{k=1}^{N} \frac{P_k}{C_{k,N+1} \cdot g^{opt}(\sigma^2)} \right), \tag{27}$$

where, $C_{k,N+1}$ is the link cost defined as in Eq. (11).

Proof: We prove it by contradiction. In the optimal flow and routing solution for the network lifetime maximization problem stated in Eq. (23, 24, 25) for homogeneous wireless sensor networks, assume there is a multi-hop sub flow η with data volume S, generated at sensor i_0 and transmitted to the fusion center through sensors $i_1, \cdots, i_T$ sequentially, i.e.,

$$S_{i_0}^{\eta} = f_{i_0,i_1}^{\eta} = f_{i_1,i_2}^{\eta} = \cdots = f_{i_{T-1},i_T}^{\eta} = f_{i_T,N+1}^{\eta} = S \quad (28)$$

then, remove this multi-hop sub flow η and add a serial of single-hop sub flow $\xi_0, \cdots, \xi_T$ as follows:

$$\begin{array}{l} S_{i_t}^{\xi_t} = f_{i_t,N+1}^{\xi_t} = \frac{C_{i_t,i_{t+1}}}{C_{i_t,N+1}} \cdot S, \qquad \forall t \in [0, T-1] \\ S_{i_T}^{\xi_T} = f_{i_T,N+1}^{\xi_T} = S. \end{array} \quad (29)$$

First, it is easy to show that both the flow conservation and energy constraints as shown in Eq. (24, 25) are satisfied by removing the sub flow η and adding the new sub flows $\xi_0, \cdots, \xi_T$.

Next, denote ϕ_0 and ϕ_1 are the objective function divided by D_r before or after removing the sub flow η and adding the new sub flows $\xi_0, \cdots, \xi_T$, i.e.,

$$\begin{array}{l} \phi_0 = \frac{1}{g^{opt}(\sigma^2)} \sum_{k=1}^{N} S_k, \\ \phi_1 = \frac{1}{g^{opt}(\sigma^2)} \left(\sum_{k=1, k \neq i_0, i_T}^{N} S_k + (S_{i_0} - S) + (S_{i_T} + S) \right) \\ \quad + \frac{1}{g^{opt}(\sigma^2)} \sum_{t=0}^{T-1} \frac{C_{i_t,i_{t+1}}}{C_{i_t,N+1}} \cdot S, \end{array} \quad (30)$$

then,

$$\phi_1 - \phi_0 = \frac{1}{g^{opt}(\sigma^2)} \sum_{t=0}^{T-1} \frac{C_{i_t,i_{t+1}}}{C_{i_t,N+1}} \cdot S \geq 0, \quad (31)$$

where, the equality holds only when the fusion center is not in the transmission range of all sensors $i_0, \cdots, i_{T-1}$, i.e., $C_{i_t,N+1} = \infty$ for all $t \in [0, T-1]$, otherwise, $\phi_1 - \phi_0 > 0$. It means that for homogeneous networks with unlimited transmission range for each sensor, single-hop routing can achieve better performance than multi-hop routing, while for homogeneous networks with limited transmission range for each sensor, single-hop routing can achieve at least as good performance as multi-hop routing.

In single-hop wireless sensor network, each sensor transmits all its measurements to the fusion center directly, and no energy is used to relay other sensors' data, thus the maximum source throughput of each sensor node is easily obtained as $S_k = P_k / C_{k,N+1}$. Therefore, according to Theorem 1, the network lifetime bound for estimation in a homogeneous network is $L \leq D_r \left(\sum_{k=1}^{N} \frac{P_k}{C_{k,N+1} \cdot g^{opt}(\sigma^2)} \right)$.

■

VI. Simulation Results

In homogeneous sensor networks, the network lifetime for estimation is maximized by optimal source coding and single-hop routing as shown in Proposition 2, while in heterogeneous sensor networks, the network lifetime for estimation is maximized by joint optimal source coding and multi-hop routing. To demonstrate the performances of the optimal coding scheme proposed in Section IV and the optimal routing scheme in Section V, we simulate a homogeneous sensor network and a heterogeneous sensor network, respectively.

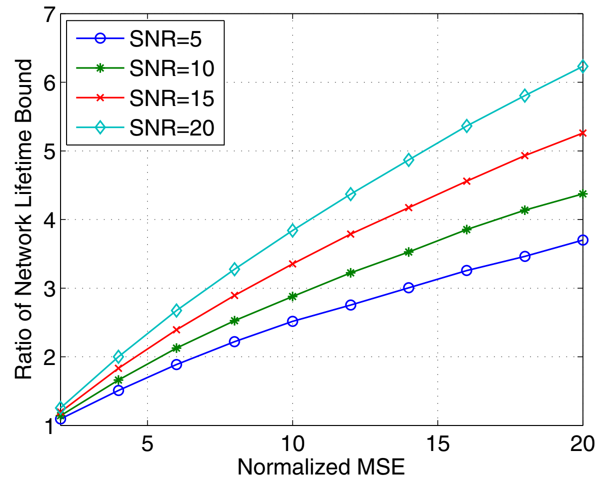


Fig. 1. Ratio of network lifetime bound for homogeneous sensor networks under different SNRs and estimation MSE requirements.

A. Homogeneous Sensor Networks

In this section, we simulate a homogeneous sensor network with $N = 500$ sensors, where the noise variance σ_k^2, the initial energy supply P_k, and the distance to the fusion center d_k for any sensor k is the same. Without loss of generality, we assume the range of the observation signal is $[-1, 1]$, i.e., $W = 1$, and path loss exponent $\alpha = 2$ (free space). Define the signal to noise ratio (SNR) as $SNR = 10 \log_{10}(W^2/\sigma^2)$. In order to demonstrate the efficiency of the proposed optimal source coding scheme, we compare the proposed algorithm with a heuristic method, where each sensor uses the same amount of energy at each estimation task period.

Denote the estimation MSE of clairvoyant estimator as $D_0 = \left(\sum_{k=1}^{N} (1/\sigma_k^2) \right)^{-1}$ and define the normalized estimation MSE requirement as $D_n = D_r / D_0$. Fig. 1 shows the ratio of network lifetime bound achieved by the proposed algorithm to that by the heuristic method under different SNRs and different normalized estimation MSE requirements. From Fig. 1, we can see that a significant gain is achieved by the proposed algorithm compared with heuristic method.

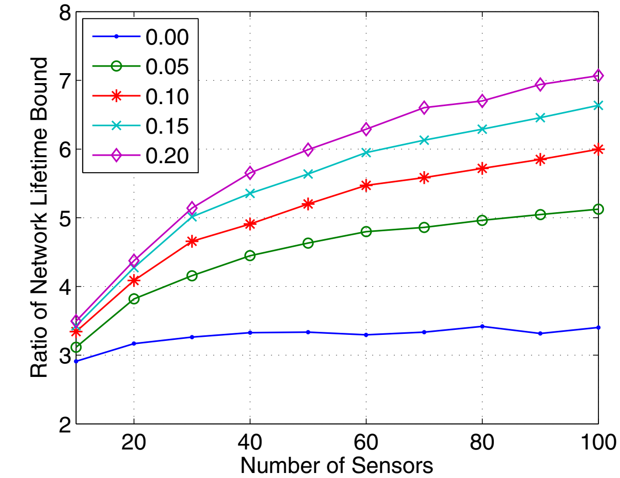
B. Heterogeneous Sensor Networks

In this section, we simulate a heterogeneous sensor network with N sensors, where the observation noise variance of each sensor is assumed to be

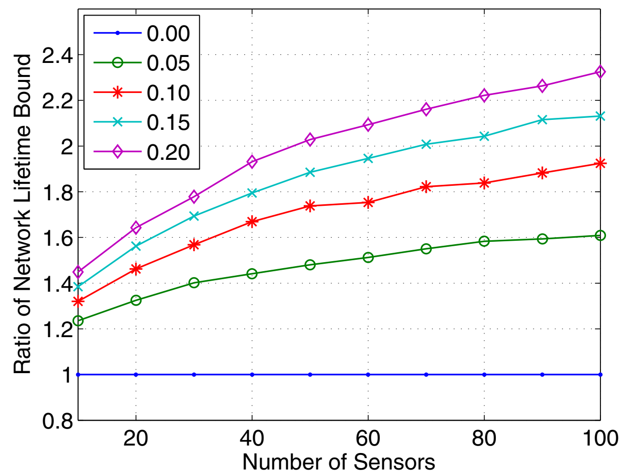
$$\sigma_k^2 = \beta + \gamma z_k, \qquad k = 1, \cdots, N, \quad (32)$$

where β models the network-wide noise variance threshold, γ controls the underlying variation from sensor to sensor, and $z_k \sim \chi_1^2$ is a Chi-Square distributed random variable with one degree of freedom. It is noted the network is homogeneous for the special case of $\gamma = 0$. In the experiments, we assume $\beta = 0.01$ and $\gamma = 0.00, 0.05, 0.10, 0.15$, or 0.20. Assume all sensors are independently and uniformly distributed in a rectangular region of $[-5, 5, -5, 5]$, and the fusion center is located at the central point of the region, i.e., $(0, 0)$. And the initial energy is still assumed to be the same for all sensors. Assume the estimation MSE requirement is $D_r = 5D_0$. To demonstrate the efficiency of the proposed algorithms, we compare it with two heuristic methods: (i) *Heuristic-I*: single-hop routing with uniform energy scheduling for each sensor, and (ii) *Heuristic-II*: single-hop routing with optimal source coding and energy scheduling.

Fig. 2(a) and Fig. 2(b) show the ratio of network lifetime bound achieved by the proposed algorithm to that by the *Heuristic-I* and *Heuristic-II* methods under different total number of sensors and different sensor noise variation parameters γ, respectively. From Fig. 2(a) and Fig. 2(b), we can see that the proposed algorithms improve the network lifetime bound significantly compared with both *Heuristic-I* and *Heuristic-II* methods. It is noted that both the optimal method and the *Heuristic-II* method use optimal source coding, and the only difference is that optimal multi-hop routing is used by the optimal solution, while single-hop routing is used by the *Heuristic-II* method. From Fig. 2(b), we see that the *Heuristic-II* method is also optimal when $\gamma = 0.00$, which confirms our conclusion in Proposition 2 that single-hop routing maximizes the network lifetime bound for homogeneous networks. From Fig. 2(b), we also can see that optimal multi-hop routing improves the network lifetime bound significantly compared with single-hop routing for heterogeneous networks. Furthermore, the gain is more significant when the network is denser since there are more opportunities for multi-hop routing, also the gain is more significant when the observation noise variances are more diverse, i.e., γ becomes bigger.



(a)



(b)

Fig. 2. Ratio of network lifetime bound by the proposed algorithm to that by two heuristic methods under different total number of sensors and different sensor noise variation parameters ($\gamma = 0.00, 0.05, 0.10, 0.15, 0.20$): (a) compared with *Heuristic-I*, (b) compared with *Heuristic-II*.

VII. Conclusions

In this paper, we consider the lifetime-distortion issue for estimation in multi-hop wireless sensor networks. First, a notion of function-based network lifetime is introduced, based on which the network lifetime maximization problem is formulated as a nonlinear programming (NLP) problem, then the NLP problem is decoupled into a source coding optimization problem and a linear programming (LP) problem for routing optimization, which are solved respectively.

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