Function-Based Network Lifetime for Estimation in Wireless Sensor Networks

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***Abstract*—Network lifetime is a critical concern in the design of wireless sensor networks. Though many different definitions of network lifetime have been used in the literature, we introduce a function-based network lifetime definition, which focuses on whether the network can perform a given task instead of whether any individual sensor is dead. Then we derive an upper bound of function-based network lifetime for estimation, and we maximize it by introducing a concept of equivalent unit-resource mean square error (MSE) function. The proposed algorithm is optimal and the simulation results show that a significant gain is achieved by the proposed algorithm compared with heuristic methods.**

***Index Terms*—Best unbiased linear estimation, distributed estimation, network lifetime, wireless sensor networks.**

I. Introduction

A COMMON goal in most wireless sensor network (WSN) applications is to reconstruct the underlying physical phenomenon, e.g., temperature, based on sensor observations. Distributed estimation of unknown deterministic parameters by a set of distributed sensor nodes and a fusion center (FC) has become an important topic in signal processing research for sensor networks. Subject to severe bandwidth and energy constraints, each sensor is allowed to transmit only a quantized version of its raw measurement to the fusion center that generates a final estimation. Recently, several bandwidth-constrained distributed estimation algorithms have been investigated [1]–[4]. In [1], a class of maximum likelihood estimators (MLEs) was proposed to attain a variance that is close to the clairvoyant estimator when the observations are quantized to one bit. The work of [2] and [3] proposed several universal decentralized estimation systems based on best linear unbiased estimation (BLUE) rule without the knowledge of noise distribution. The work of [4] proposed optimal distributed estimation algorithms to minimize the estimation mean square error (MSE) under the total rate constraint. Also energy-constrained distributed estimation has been studied in [5]–[8]. In [5] and [6], the total sensor transmission energy is minimized by selecting the optimal quantization levels while meeting the target estimation MSE requirements. On the contrary, the work of [7] and [8] is to minimize the estimation MSE under the total energy constraints.

However, to the best of our knowledge, the network lifetime analysis, which is a critical concern in the design of wireless sensor networks, for estimation application in wireless sensor networks is not yet available in the literature. Therefore, in this letter, we study how to optimize the network lifetime which is defined as *the estimation task cycles successfully accomplished until the network cannot perform the task with a given distortion requirement any more*.

II. Estimation in Wireless Sensor Networks

Consider a dense sensor network that includes N distributed sensors and a fusion center to estimate the unknown parameter θ, where each sensor can observe, quantize, and transmit its observation to the fusion center that makes the final estimation based on the received messages. Assume the sensor observation is corrupted by additive noise and is described by

$$x_k = \theta + n_k, k = 1, \cdots, N \tag{1}$$

where the observation noise of all sensors n_k are assumed to be zero mean, spatially uncorrelated with variance σ_k^2, otherwise unknown.

Subject to severe bandwidth and energy constraints, each sensor in wireless sensor networks transmits only a quantized version of its raw measurement to the fusion center, i.e., $m_k = Q_k(x_k)$, where $Q_k(x_k)$ is a quantization function. Assume there are K received observations $(m_1, \cdots, m_K)$ at the fusion center, then the fusion center makes an estimation of θ using a fusion function $f : \bar{\theta} = f(m_1, \cdots, m_K)$. The quality of an estimation for θ is measured by the MSE criterion.

Assume the observation signal is bounded, i.e., $x_k \in [-W, W]$, we adopt a probabilistic quantization scheme [5] at each sensor to make the local quantization. Suppose all the observations of K active sensors $x_k (k = 1, \cdots, K)$ are quantized into b_k-bits discrete messages $m_k(x_k, b_k)$, respectively, with the probabilistic quantization scheme, then the variance of the quantized message is $E(m_k(x_k, b_k) - \theta)^2 \le \sigma_k^2 + \delta_k^2(b_k) := \pi_k^2(\sigma_k^2, b_k)$, where $\delta_k^2(b_k) = W^2/(2^{b_k} - 1)^2$ denotes the upper bound of the quantization noise variance.

Treating the received quantization messages m_k as the new measurements of the unknown parameter θ, the fusion center makes the final estimation of θ using a quasi-BLUE (best linear unbiased estimate) scheme [5] as follows:

$$\bar{\theta} = \left(\sum_{k=1}^{K} \frac{1}{\pi_k^2(\sigma_k^2, b_k)} \right)^{-1} \sum_{k=1}^{K} \frac{m_k}{\pi_k^2(\sigma_k^2, b_k)}. \tag{2}$$

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Notice that $\bar{\theta}$ is an unbiased estimator of θ since every m_k is unbiased. Moreover, the estimation MSE of the quasi-BLUE estimator is

$$E(\bar{\theta}-\theta)^2 \leq \left(\sum_{k=1}^{K} \frac{1}{\pi_k^2(\sigma_k^2, b_k)}\right)^{-1}. \tag{3}$$

III. Network Lifetime for Estimation

In the literature, many different lifetime definitions are used, such as duration of time until the first sensor failure due to battery depletion in [9], fraction of surviving nodes in a network in [10] and [11], and mean expiration time in [12] etc. However, these notions of network lifetime mainly focus on the time until the first node or a fraction of nodes deplete, even though the remaining network may be still functional from the application perspective. In this letter, we introduce a notion of function-based network lifetime, which focuses on whether the network can perform a given task instead of whether any individual sensor is dead.

Definition 1 (Function-Based Network Lifetime): For estimation application, the network is considered functional if it can produce an estimation satisfying a given distortion requirement D_r; otherwise, it is nonfunctional. The network lifetime L is defined as the estimation cycles accomplished before the network becomes nonfunctional because of sensor depletion.

At different estimation cycles, the parameter θ is assumed to be unrelated, and the estimation at each cycle is performed independently using only the observations made by all sensors in the given estimation cycle. Based on the estimation system model in Section II, assume a sensor network with N sensors, each with observation noise variance $\sigma_k^2 (k = 1, \cdots, N)$. To satisfy the given estimation distortion requirement D_r, at each estimation cycle, a subset of the sensors is required to observe the parameter θ and transmit their quantized measurements to the fusion center to make the final estimation.

Proposition 1: Assume sensor $k (k = 1, \cdots, N)$ can make a total of M_k measurements and quantize its measurements using probabilistic quantization scheme to $b_{k,i} (i = 1, \cdots, M_k)$ bits before it depletes. Then the function-based network lifetime L for estimation application is bounded as follows:

$$L \leq D_r \left(\sum_{k=1}^{N} \sum_{i=1}^{M_k} \frac{1}{\pi_k^2(\sigma_k^2, b_{k,i})}\right) \tag{4}$$

where M_k and $b_{k,i}$ are variables to be determined based on each sensor's energy resource $P_k (k = 1, \cdots, N)$.

Proof: At each estimation cycle $l \in [1, L]$, denote the subset of observations each sensor k makes and sends to the fusion center is $O_{k,l}$. Then for any sensor $k \in [1, N]$

$$\begin{aligned} & O_{k,i} \cap O_{k,j} = \emptyset, \forall i, j \in [1, L], \text{ and } i \neq j \\ & \bigcup_{l=1}^{L} O_{k,l} \subseteq \{1, \cdots, M_k\} \end{aligned} \tag{5}$$

and for any estimation cycle $l \in [1, L]$, we have

$$\left(\sum_{k=1}^{N} \sum_{i \in O_{k,l}} \frac{1}{\pi_k^2(\sigma_k^2, b_{k,i})}\right)^{-1} \leq D_r. \tag{6}$$

So

$$\sum_{l=1}^{L} \sum_{k=1}^{N} \sum_{i \in O_{k,l}} \frac{1}{\pi_k^2(\sigma_k^2, b_{k,i})} \geq \frac{L}{D_r} \tag{7}$$

i.e.,

$$\sum_{k=1}^{N} \sum_{i=1}^{M_k} \frac{1}{\pi_k^2(\sigma_k^2, b_{k,i})} \geq \frac{L}{D_r} \tag{8}$$

therefore

$$L \leq D_r \left(\sum_{k=1}^{N} \sum_{i=1}^{M_k} \frac{1}{\pi_k^2(\sigma_k^2, b_{k,i})}\right). \tag{9}$$

■

It is noted that the upper bound shown in Proposition 1 could be closely approached by appropriately scheduling the subset of active sensors in each estimation cycle such that the actual estimation MSE obtained is equal to or slightly smaller than D_r in (6).

Based on the estimation system model and the definition of function-based network lifetime, the objective of this letter is to maximize the function-based network lifetime bound shown in (4) under the energy resource constraint of each sensor, i.e.,

$$\begin{aligned} & \max D_r \left(\sum_{k=1}^{N} \sum_{i=1}^{M_k} \frac{1}{\pi_k^2(\sigma_k^2, b_{k,i})}\right) \\ & \text{s.t. } \sum_{i=1}^{M_k} e_k(b_{k,i}) \leq P_k, \forall k \in [1, N] \\ & e_k(b_{k,i}) \leq p_k, \forall k \in [1, N], i \in [1, M_k] \end{aligned} \tag{10}$$

where P_k is the total energy resource of sensor k, p_k is the individual maximum energy constraint of sensor k for each observation, $e_k(b_{k,i})$ is the transmission energy cost for sensor k to transmit a $b_{k,i}$-bit quantization message to the fusion center, and $M_k \geq 0$ and $b_{k,i} \geq 0$ defined as before are variables to be optimized. To facilitate the solution to (10), we first introduce a concept of equivalent unit-resource MSE function.

Definition 2 (Equivalent Unit-Resource MSE Function): For a quantized message from a sensor with observation noise variance σ^2 and quantization bit rate b, the estimation variance is $\pi^2(\sigma^2, b) := \sigma^2 + (W^2)/((2^b - 1)^2)$ as shown in Section II. Denote the resource cost by this message as $r(b)$. Then, the equivalent unit-resource MSE function is defined as

$$g(\sigma^2, b) := r(b) \cdot \pi^2(\sigma^2, b) = r(b) \cdot \left(\sigma^2 + \frac{W^2}{(2^b - 1)^2}\right). \tag{11}$$

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Based on this definition, from the estimation MSE aspect, a sensor with quantization bit rate b, resource cost $r(b)$, and estimation MSE $\pi^2(\sigma^2, b)$ can be treated as $r(b)$ equivalent unit-resource sensors, each with the same estimation MSE $g(\sigma^2, b)$. That is why $g(\sigma^2, b)$ is called equivalent unit-resource MSE function. It is worth to note that this definition is quite generic, where the resource can be bandwidth, energy, etc.

Here, we focus on the energy resource, i.e., let $r(b)$ in (11) be the transmission energy cost $e(b)$. We consider two different transmission models: 1) binary transmission and 2) quadrature amplitude modulation (QAM)-based transmission. Assume the transmission distance from sensor k to the fusion center is d_k, and the channel power attenuation factor is $a_k = d_k^{\alpha}$, where α is the path loss exponent. Then, to reliably transmit b_k bits message from sensor k to the fusion center, the transmission energy cost for the binary transmission model, where each bit will be transmitted separately, is

$$e_1(b_k) = c_1 \cdot a_k \cdot b_k \tag{12}$$

where c_1 is a system constant. To minimize the transmission bandwidth and transmission delay, the b_k bits can be transmitted simultaneously using quadrature amplitude modulation (QAM) with constellation size 2^{b_k}, and then the transmission energy cost [13] is given by

$$e_2(b_k) = c_2 \cdot a_k \cdot (2^{b_k} - 1) \tag{13}$$

where c_2 is a system constant. For both energy models above, it can be shown that the corresponding equivalent unit-resource MSE functions $g(\sigma^2, b)$ defined in (11) are convex over b.

Based on the convexity of $g(\sigma^2, b)$, we further define the optimal unit-resource MSE function $g^{\rm opt}(\sigma^2)$, and the corresponding optimal quantization bit rate $b^{\rm opt}(\sigma^2)$ and optimal transmission energy $e^{\rm opt}(\sigma^2)$ as follows:

$$\begin{aligned} b^{\rm opt}(\sigma^2) &= \arg\min_{b\in\mathbb{Z}^+, e(b)\le p} g(\sigma^2, b) \\ g^{\rm opt}(\sigma^2) &= \min_{b\in\mathbb{Z}^+, e(b)\le p} g(\sigma^2, b) = g(\sigma^2, b^{\rm opt}(\sigma^2)) \\ e^{\rm opt}(\sigma^2) &= e(b^{\rm opt}(\sigma^2)) \end{aligned} \tag{14}$$

where p denotes the individual maximum energy constraint. It is noted that the minimization in (14) involves just a simple one-dimensional numerical search over $b \in \mathbb{Z}^+$.

Theorem 1: The bound of function-based network lifetime for estimation is

$$\begin{aligned} L &\le D_r \left(\sum_{k=1}^{N} \frac{P_k}{g_k^{\rm opt}(\sigma_k^2)} \right) \\ &= D_r \left(\sum_{k=1}^{N} \frac{P_k}{e_k^{\rm opt}(\sigma_k^2) \cdot \pi_k^2(\sigma_k^2, b_k^{\rm opt}(\sigma_k^2))} \right) \end{aligned} \tag{15}$$

where $N, \sigma_k^2, M_k, b_{k,i}$, and D_r are defined as before, and $g_k^{\rm opt}(\sigma_k^2), b_k^{\rm opt}(\sigma_k^2)$ and $e_k^{\rm opt}(\sigma_k^2)$ are the optimal unit-resource MSE function, optimal quantization bit rate, and optimal transmission energy per observation, of sensor k, respectively.

Proof: Assume sensor k makes M_k measurements, each with quantization bit rate $b_{k,i}$ and transmission energy cost $e_k(b_{k,i})$, before the sensor depletes, i.e., $\sum_{i=1}^{M_k} e_k(b_{k,i}) \le P_k$. Then as shown in (4), the network lifetime bound is

$$L \le D_r \left(\sum_{k=1}^{N} \sum_{i=1}^{M_k} \frac{1}{\pi_k^2(\sigma_k^2, b_{k,i})} \right). \tag{16}$$

According to the definition of $g(\sigma^2, b)$ and $g^{\rm opt}(\sigma^2)$ and the energy constraints in (10)

$$\begin{aligned} L &\le D_r \left(\sum_{k=1}^{N} \sum_{i=1}^{M_k} \frac{1}{\pi_k^2(\sigma_k^2, b_{k,i})} \right) \\ &= D_r \left(\sum_{k=1}^{N} \sum_{i=1}^{M_k} \frac{e_k(b_{k,i})}{g_k(\sigma_k^2, b_{k,i})} \right) \\ &\le D_r \left(\sum_{k=1}^{N} \sum_{i=1}^{M_k} \frac{e_k(b_{k,i})}{g_k^{\rm opt}(\sigma_k^2)} \right) \\ &\le D_r \left(\sum_{k=1}^{N} \frac{P_k}{g_k^{\rm opt}(\sigma_k^2)} \right) \end{aligned} \tag{17}$$

thus, the theorem is proved. ■

Note that the equality in (17) is achieved when each sensor node adopts optimal quantization bit rate $b^{\rm opt}(\sigma^2)$ and optimal transmission energy $e^{\rm opt}(\sigma^2)$ to quantize and transmit its observations. As shown before, the optimal quantization bit rate $b^{\rm opt}(\sigma^2)$, and optimal transmission energy $e^{\rm opt}(\sigma^2)$ of each sensor can be easily obtained by minimizing its equivalent unit-resource MSE function, which only depends on its own observation noise variance and transmission system parameters; therefore, this optimization can be done in a completely distributed manner.

IV. Simulation Results

In this section, we simulate a homogeneous and a heterogeneous networks and apply the proposed algorithm to determine their network lifetime bounds. All the simulation results are obtained by repeating the experiments for 10 000 times and averaging the corresponding results.

A. Homogeneous Sensor Networks

In this section, we simulate a homogeneous sensor network with $N = 500$ sensors, where the noise variance σ_k^2, the initial energy P_k, and the distances to the fusion center d_k for all sensors are the same. Without loss of generality, we assume $d_k = 1$, the normalized initial energy $P_k/c = 10\,000$, the range of the observation signal is $[-1, 1]$, i.e., $W = 1$, and path loss exponent $\alpha = 2$ (free space). Define the signal-to-noise ratio (SNR) as $\mathrm{SNR} = 10\log_{10}(W^2/\sigma^2)$ and generate different SNR by changing the observation noise variance σ^2. In order to demonstrate the efficiency of the proposed method, we compare the proposed algorithm with a heuristic method, where each sensor uses the same amount of energy to achieve the distortion requirement at each estimation task period; thus, all the sensors will deplete at the same time.

Denote the estimation MSE of clairvoyant estimator as $D_0 = (\sum_{k=1}^{N}(1/\sigma_k^2))^{-1}$ and define the normalized estimation MSE requirement as $D_n = D_r/D_0$. Fig. 1(a) and (b) shows the ratio of network lifetime bound by the proposed algorithm to

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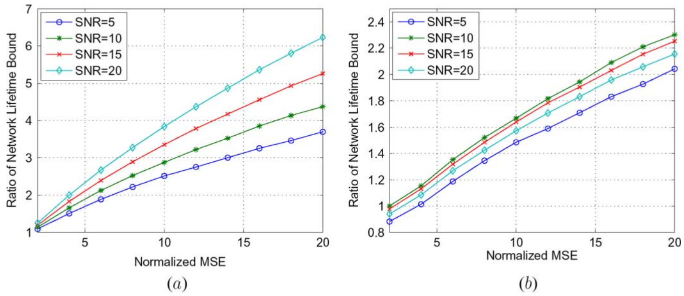

Fig. 1. Ratio of network lifetime bound for homogeneous sensor networks. (a) Binary model. (b) QAM model.

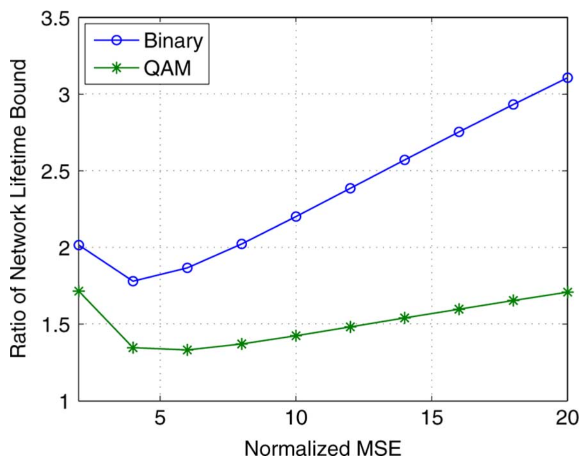

Fig. 2. Ratio of network lifetime bound for heterogeneous sensor networks with $\beta = 0.01, \gamma = 0.1$ under different transmission models.

that by the heuristic method under different SNRs and different normalized estimation MSE requirements using the binary and QAM-based transmission models, respectively. From Fig. 1(a) and (b), we can see that a significant gain on network lifetime is achieved by the proposed algorithm compared with heuristic method for both energy models, and the gain for binary model is larger than the gain for QAM model.

B. Heterogeneous Sensor Networks

In this section, we simulate a heterogeneous sensor network with $N = 100$ sensors, where the observation noise variance of each sensor is assumed to be

$$\sigma_k^2 = \beta + \gamma z_k, k = 1, \cdots, N \tag{18}$$

where β models the network-wide noise variance threshold, γ controls the underlying variation from sensor to sensor, and $z_k \sim \chi_1^2$ is a Chi-Square distributed random variable with one degree of freedom. In the experiments, we assume $\beta = 0.01$ and $\gamma = 0.1$. Assume the distance from each sensor to the fusion center is independently and uniformly distributed from 1 to 5, i.e., $d_k \sim U[1, 5]$. Also the initial energy is still assumed to be the same for all sensors. Fig. 2 shows the ratio of network lifetime bound achieved by the proposed algorithm to that by the heuristic method under different normalized estimation MSE requirements and two different transmission models. From Fig. 2, similar conclusions as in the homogeneous network case can be drawn.

V. CONCLUSION

In this letter, we considered the function-based network lifetime for estimation application in energy-limited wireless sensor networks. First, we derived the upper bound of function-based network lifetime. Then, to optimize the function-based network lifetime bound, we proposed a concept of equivalent unit-resource MSE function, which is generic and can be used to analyze different models. The proposed algorithm is optimal, and the simulation results show that a significant gain is achieved by the proposed algorithm compared with heuristic methods. The extension to multihop wireless sensor network is being investigated in our current work. It is also noted that the concept of function-based network lifetime is quite generic and could be extended to a broad range of WSN applications, which is another interesting direction for future work.

REFERENCES

[1] A. Ribeiro and G. Giannakis, "Bandwidth-constrained distributed estimation for wireless sensor networks, part i: Gaussian case," *IEEE Trans. Signal Process.*, vol. 54, no. 3, pp. 1131–1143, Mar. 2006.
[2] Z.-Q. Luo, "Universal decentralized estimation in a bandwidth constrained sensor network," *IEEE Trans. Inf. Theory*, vol. 51, no. 6, pp. 2210–2219, June 2005.
[3] Z.-Q. Luo and J.-J. Xiao, "Decentralized estimation in an inhomogeneous sensing environment," *IEEE Trans. Inf. Theory*, vol. 51, no. 10, pp. 3564–3575, Oct. 2005.
[4] J. Li and G. AlRegib, "Rate-constrained distributed estimation in wireless sensor networks," *IEEE Trans. Signal Process.*, vol. 55, no. 5, pp. 1634–1643, May 2007.
[5] J.-J. Xiao, S. Cui, Z.-Q. Luo, and A. Goldsmith, "Power scheduling of universal decentralized estimation in sensor networks," *IEEE Trans. Signal Process.*, vol. 54, no. 2, pp. 413–422, Feb. 2006.
[6] A. Krasnopeev, J.-J. Xiao, and Z.-Q. Luo, "Minimum energy decentralized estimation in sensor network with correlated sensor noise," *EURASIP J. Wireless Commun. Netw.*, vol. 5, no. 4, pp. 473–482, 2005.
[7] V. Aravinthan, S. Jayaweera, and K. Tarazi, "Distributed estimation in a power constrained sensor network," in *Proc. IEEE 63rd Vehicular Technology Conf.*, May 7–10, 2006, vol. 3, pp. 1048–1052.
[8] J. Li and G. AlRegib, "Energy-constrained distributed estimation in wireless sensor networks," in *Proc. Military Communications Conf.*, Orlando, FL, Oct. 29–31, 2007.
[9] J. Chang and L. Tassiulas, "Energy conserving routing in wireless ad-hoc networks," in *Proc. IEEE INFOCOM*, Tel Aviv, Israel, Mar. 2000.
[10] R. Wattenhofer, L. Li, P. Bahl, and Y. Wang, "Distributed topology control for power efficient operation in multihop wireless ad hoc networks," in *Proc. IEEE INFOCOM*, Anchorage, AK, Apr. 2001, pp. 1388–1397.
[11] Y. Xu, J. Heidemann, and D. Estrin, "Geography-informed energy conservation for ad hoc routing," in *Proc. ACM SIGMOBILE 7/01*, Rome, Italy, 2001, pp. 70–84.
[12] C.-K. Toh, "Maximum battery life routing to support ubiquitous mobile computing in wireless ad hoc networks," *IEEE Commun. Mag.*, vol. 39, no. 6, pp. 138–147, Jun. 2001.
[13] S. Cui, A. J. Goldsmith, and A. Bahai, "Energy-constrained modulation optimization," *IEEE Trans. Wireless Commun.*, vol. 4, no. 5, pp. 2349–2360, Sep. 2005.

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