Network Lifetime Maximization for Estimation in Multihop Wireless Sensor Networks

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***Abstract*—We consider the distributed estimation by a network consisting of a fusion center and a set of sensor nodes, where the goal is to maximize the network lifetime, defined as the estimation task cycles accomplished before the network becomes nonfunctional. In energy-limited wireless sensor networks, both local quantization and multihop transmission are essential to save transmission energy and thus prolong the network lifetime. The network lifetime optimization problem includes three components: *i*) optimizing source coding at each sensor node, *ii*) optimizing source throughput of each sensor node, and *iii*) optimizing multihop routing path. Fortunately, source coding optimization can be decoupled from source throughput and multihop routing path optimization, and is solved by introducing a concept of *equivalent 1-bit MSE function*. Based on the optimal source coding, the source throughput and multihop routing path optimization is formulated as a linear programming (LP) problem, which suggests a new notion of *character-based routing*. The proposed algorithm is optimal and the simulation results show that a significant gain is achieved by the proposed algorithm compared with heuristic methods.**

***Index Terms*—Best linear unbiased estimation (BLUE), distributed estimation, distributed signal processing, multihop wireless sensor networks, network lifetime.**

I. Introduction

WIRELESS sensor networks (WSN), consisting of a large number of geographically distributed sensor nodes, have many current and future envisioned applications, such as environment monitoring, battlefield surveillance, health care, and home automation. Though each sensor is characterized by low power constraint and limited computation and communication capabilities due to various design considerations such as small size battery, bandwidth and cost, potentially powerful networks can be constructed to accomplish various high-level tasks via sensor cooperation [1], such as distributed estimation, distributed detection, and target localization and tracking.

A common goal in most WSN applications is to reconstruct the underlying physical phenomenon, e.g., temperature, based on sensor measurements. Distributed estimation of unknown deterministic parameters by a set of distributed sensor nodes and a fusion center has become an important topic in signal processing research for wireless sensor networks [2], where sensor nodes collect real-valued data, perform a local data compression, and send the resulting messages to the fusion center, while the fusion center combines the received messages to produce a final estimation of the observed parameter. Most of the early works [3]–[7] assume that the joint distribution of sensors' observations is known and that the real-valued messages can be sent from the sensors to the fusion center without distortion, which are unrealistic for practical sensor networks because of the high communication and high energy cost.

Subject to the resource (bandwidth and energy) limitation nature of wireless sensor networks, several bandwidth-constrained distributed estimation algorithms [8]–[20] have been investigated recently. The work of [8]–[10] addressed various design and implementation issues to digitize the transmitted signal into one or several binary bits using the joint distribution of sensors' data. In [11] and [12], a class of maximum likelihood estimators (MLE) was proposed to attain a variance that is close to the clairvoyant estimator when the observations are quantized to one bit. The work of [13] and [14] addressed the maximum likelihood estimation over noisy channel for bandwidth-constrained sensor networks with or without knowing the sensing and channel noise parameters at the fusion center. Without the knowledge of noise distribution, the work of [15] and [16] proposed to use a training sequence to aid the design of local data quantization strategies, and the work of [17] and [18] proposed several universal (pdf-unaware) decentralized estimation systems based on best linear unbiased estimation (BLUE) rule for distributed parameter estimation in the presence of unknown, additive sensor noise. While most of the aforementioned work on bandwidth-constrained distributed estimation are posed for a given number of sensors (one observation per sensor) [8]–[18], the work of [19] proposed quasi-optimal distributed parameter estimation algorithms to minimize the estimation mean square error (MSE) with a total rate constraint. Bandwidth-constrained distributed estimation in encrypted wireless sensor networks is also addressed in [20].

To explicitly address the energy constraint in wireless sensor networks, the minimal-energy distributed estimation problem has also been recently considered in [21]–[26]. In [21] and [22], the total sensor transmission energy is minimized by selecting the optimal quantization levels while meeting the target estimation MSE requirements. On the contrary, the work of [23], [24] is to minimize the estimation MSE under the given energy constraints. The work of [25], [26] addressed the energy-constrained distributed estimation problem (under the BLUE fusion rule) by exploiting long-term noise variance statistics.

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Decentralized estimation using ad hoc WSNs is also recently addressed, which is based on successive refinements of local estimates maintained at individual sensors. At each iteration, the sensors exchange quantized messages with their immediate neighbors, then each sensor uses this information to refine its local estimate. In this context, decentralized estimation of deterministic parameters in linear data models was considered in [27]–[33] using the notion of consensus averaging. Decentralized estimation of parameter vectors in general (possibly nonlinear and/or non-Gaussian) data models was considered in [34], [35] using both MLE and BLUE schemes.

All the aforementioned algorithms address distributed estimation problem from either *rate-distortion* perspective or *energy-distortion* perspective. However, they are not necessarily optimal in the sense of network lifetime, which is a critical concern in the design of wireless sensor networks [36]. To the best of our knowledge, the network lifetime issue for distributed estimation applications in wireless sensor networks has not yet been addressed explicitly in the literature. In this paper, we study the *lifetime-distortion* issue of the estimation application in energy-limited sensor networks, where the lifetime is defined as *the estimation task cycles successfully accomplished until the network can not perform the task with a given distortion requirement any more*.

In energy-limited wireless sensor networks, both local quantization and multihop transmission are essential to save transmission energy and thus prolong the network lifetime. To maximize the network lifetime for the estimation application, three factors are needed to be optimized together: *i) source coding, i.e., quantization level of each observation, ii) source throughput, i.e., total number of observations or total information bits generated by each sensor, and iii) multihop routing path to transmit the observations from all sensors to the fusion center*. This problem can be formulated as a nonlinear programming (NLP) problem. Further, as we will show in this paper, source coding optimization can be decoupled from source throughput and multihop routing optimization and solved by introducing a concept of equivalent 1-bit MSE function. It is noted that the proposed algorithm determines the optimal quantizer locally at each sensor without knowing other sensors' information, thus it can be implemented in a distributed manner. On the other hand, the source throughput and multihop routing needs to be optimized jointly and it can be formulated as a linear programming (LP) problem [37] based on the optimal source coding. It is interesting to see that the solution implies a *character-based routing*, where a sensor node only relays other sensors' observations that are more accurate than its own observations, which is different from the traditional distance-based routing, where sensor nodes closer to the fusion center relay information for sensor nodes farther away from the fusion center.

The rest of the paper is organized as follows. Section II introduces the system model of estimation in multihop wireless sensor networks. Section III formulates the network lifetime bound maximization problem for the estimation application in wireless sensor networks as a nonlinear programming (NLP) problem, and then decouples the original problem into two subproblems without compromising the optimality. In Sections IV and V, we solve the two subproblems, i.e., *i*) source coding optimization, and *ii*) joint source throughput and multihop routing optimization, respectively. Section VI gives some simulation results that demonstrate the efficiency of the proposed algorithms. Finally, conclusions are given in Section VII.

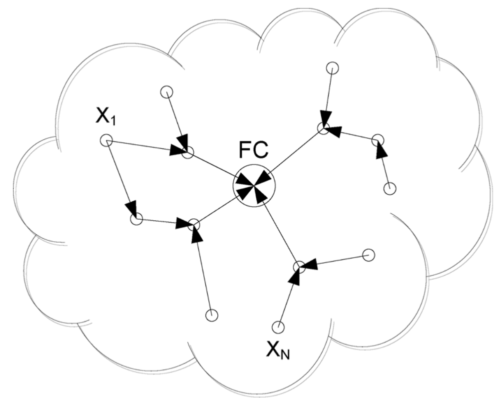

Fig. 1. An example of a wireless sensor network with N distributed sensor nodes. Each sensor can observe the phenomenon, quantize and transmit its observation to the fusion center (FC) via multihop wireless channel, and the fusion center makes the final estimation based on all the received messages. In directed solid lines, a chosen multihop routing path is shown, where the data from a sensor can be relayed by multiple sensors, meanwhile a sensor can relay data for multiple sensors.

II. System Model and Preliminaries

We consider a dense sensor network including N distributed sensor nodes and a fusion center, denoted as node $N+1$, to observe and estimate an unknown parameter θ. An example network is shown in Fig. 1.

A. System Model

First, each sensor k can make observations on the unknown parameter θ. The observations are corrupted by additive noise and described by

$$x_k = \theta + n_k, \qquad k = 1, \ldots, N. \tag{1}$$

We assume that the observation noises of all sensors n_k ($k = 1, \ldots, N$) are zero mean, spatially uncorrelated with variance σ_k^2, while the noise at each sensor is assumed to be temporally independent and identically distributed (i.i.d.), otherwise unknown. By a suitable linear scaling, the above data model in (1) is equivalent to the one where sensors observe θ with different attenuations, namely, $x_k = h_k\theta + n_k$. Indeed, if we let $x'_k = x_k/h_k$ and $n'_k = n_k/h_k$, then $x'_k = \theta + n'_k$, which is identical to that in (1) with equivalent noise variance σ_k^2/h_k^2.

Subject to severe bandwidth and energy limitations, each sensor is prevented from transmitting real-valued (analogy) data to the fusion center, that is to say, a local quantization $m_k = Q_k(x_k)$ is performed before transmission, where $Q_k(x_k)$ is a quantization function, and only the quantization message m_k is transmitted to the fusion center via multihop wireless channel.

Assume there are K received observations $(m_1, m_2, \ldots, m_K)$ at the fusion center, then the fusion center produces a final estimation of θ by combining all the available observations using a fusion function $f : \bar{\theta} = f(m_1, m_2, \ldots, m_K)$. The quality of an estimation for θ is measured by the mean square error (MSE) criterion.

B. Blue Estimation Rule

If the fusion center has the knowledge of the sensor noise variance σ_k^2 $(k = 1, \ldots, K)$ and the sensors can perfectly send their observations x_k $(k = 1, \ldots, K)$ to the fusion center, the BLUE estimator [38] for θ is known to be

$$\bar{\theta} = \left(\sum_{k=1}^{K} \frac{1}{\sigma_k^2}\right)^{-1} \sum_{k=1}^{K} \frac{x_k}{\sigma_k^2} \tag{2}$$

and the estimation MSE of the BLUE estimator is

$$E(\bar{\theta} - \theta)^2 = \left(\sum_{k=1}^{K} \frac{1}{\sigma_k^2}\right)^{-1}. \tag{3}$$

But the BLUE scheme is impractical for wireless sensor networks because of high communication cost (bandwidth and energy). Instead of sending the real-valued observations to the fusion center directly, quantization at the local sensors is essential to reduce the communication cost. In this paper, we adopt a probabilistic quantization scheme [21], [32], [33] at each sensor to make the local quantization, as well as a quasi-BLUE estimation scheme at the fusion center to make the final estimation.

Suppose the observation x is bounded to $[-W, W]$, that is, $x = \theta + n \in [-W, W]$. The probabilistic quantization with b bits is summarized as follows: uniformly divide $[-W, W]$ into intervals of length $\Delta = 2W/(2^b - 1)$, and round x to the neighboring endpoints of these small intervals in a probabilistic manner. More specifically, suppose $-W + i\Delta \le x \le -W + (i+1)\Delta$, where $0 \le i \le 2^b - 2$, then x is quantized to $m(x, b)$ according to

$$\begin{aligned} P\{m(x,b) &= -W + i\Delta\} = 1 - r \\ P\{m(x,b) &= -W + (i+1)\Delta\} = r \end{aligned} \tag{4}$$

where $r = (x + W - i\Delta)/\Delta \in [0, 1]$. As shown in [21], the quantized message $m(x, b)$ is an unbiased estimator of θ with a variance

$$E\left(|m(x,b) - \theta|^2\right) \le \sigma^2 + \frac{W^2}{\left(2^b - 1\right)^2} := \pi^2(\sigma^2, b) \tag{5}$$

where $W^2/\left(2^b - 1\right)^2$ for $b > 0$ denotes the upper bound of the quantization noise variance.

Now suppose all the observations of the K active sensors $x_k(k = 1, \ldots, K)$ are quantized into b_k-bits discrete messages $m_k(x_k, b_k)$ respectively with the probabilistic quantization scheme. Treating all the quantized messages m_k as the new observations for the fusion center, the quasi-BLUE estimator based on the quantized message has the following form:

$$\bar{\theta} = \left(\sum_{k=1}^{K} \frac{1}{\pi_k^2(\sigma_k^2, b_k)}\right)^{-1} \sum_{k=1}^{K} \frac{m_k}{\pi_k^2(\sigma_k^2, b_k)}. \tag{6}$$

Notice that $\bar{\theta}$ is an unbiased estimation of θ because every m_k is unbiased. Moreover, the estimation MSE of the quasi-BLUE estimator is

$$E(\bar{\theta} - \theta)^2 \le \left(\sum_{k=1}^{K} \frac{1}{\pi_k^2(\sigma_k^2, b_k)}\right)^{-1}. \tag{7}$$

C. Energy Model

Assume sensor nodes can adjust their transmission power to control the transmission range. The energy consumed by sensor i to reliably transmit a b-bit message to sensor j is

$$e(b) = c \cdot b \cdot d_{i,j}^{\alpha} \tag{8}$$

where c is a system constant denoting the energy required by a transmitter amplifier to transmit 1-bit one meter, α is the path loss exponent depending on the medium properties, and $d_{i,j}$ is the distance between sensor i and sensor j.

III. Network Lifetime for Estimation

Network lifetime is a critical concern in the design of wireless sensor networks. In this section, we first define the network lifetime and then formulate the network lifetime maximization problem.

A. Function-Based Network Lifetime

In the literature, many different lifetime definitions are used, such as, duration of time until the first sensor failure due to battery depletion [39], fraction of surviving nodes in a network [40], [41], and mean expiration time [42] etc. However, these notions of network lifetime mainly focus on the time until the first node or a fraction of nodes deplete even though the remaining network may be still functional from the application perspective. In this paper, we introduce a notion of function-based network lifetime, which focuses on whether the network can perform a given task instead of whether any individual sensor is dead.

Definition 1 (Function-Based Network Lifetime): For the estimation application, the network is considered functional if it can produce an estimation satisfying a given distortion requirement D_r, otherwise it is nonfunctional. The network lifetime L is defined as the estimation task cycles accomplished before the network becomes nonfunctional, where each time when the sensor network makes an estimation is denoted as an estimation task cycle.

At different estimation cycles, the parameter θ is assumed to be unrelated, and the estimation at each cycle is performed independently using only the observations made by all sensors in the given estimation cycle. Based on the system model in Section II, assume a sensor network with N sensors, each with observation noise variance σ_k^2 $(k = 1, \ldots, N)$. To satisfy the given estimation distortion requirement D_r, at each estimation cycle, a subset of the sensors is required to observe the parameter θ and transmit their quantized measurements to the fusion center to make the final estimation.

Proposition 1: Assume sensor k $(k = 1, \ldots, N)$ makes a total of M_k measurements and quantize its measurements using probabilistic quantization scheme to $b_{k,i}$ $(i = 1, \ldots, M_k)$ bits, respectively, before it depletes. Then the function-based network lifetime L for the estimation application is bounded as follows:

$$L \le D_r \left(\sum_{k=1}^{N} \sum_{i=1}^{M_k} \frac{1}{\pi_k^2(\sigma_k^2, b_{k,i})} \right) \tag{9}$$

where N, σ_k^2, M_k, $b_{k,i}$, and D_r are defined as above, and $\pi_k^2(\sigma_k^2, b_{k,i})$ as defined in (5).

Proof: Refer to Appendix A for the complete proof of this statement. ∎

Based on the system model and the definition of function-based network lifetime, the objective of this paper is to maximize the function-based network lifetime bound shown in (9) under the energy resource constraint of each sensor.

B. Nonlinear Programming (NLP) Formulation

Model the wireless sensor network as a directed graph $G(V, E)$, where V is the set consisting of all the N sensor nodes and the fusion center (node $N+1$), i.e., $V = [1, N+1]$, E is the set of directed links in the network. An edge $(i, j) \in E$ iff $d_{i,j} \le R$, where $d_{i,j}$ is the distance between node i and node j, and R is the maximum transmission range. The link cost to transmit a unit bit information from node i to node j, denoted as $C_{i,j}$, depends on the distance $d_{i,j}$ between them based on the energy model in (8) as follows,

$$C_{i,j} = \begin{cases} c d_{i,j}^{\alpha}, & \text{if } d_{i,j} \le R \\ +\infty, & \text{otherwise} \end{cases} \tag{10}$$

where c and α are defined as before.

Assume each sensor has a limited energy supply P_k ($k = 1, \ldots, N$). During the lifetime of the network, assume sensor k makes a total of M_k measurements and quantize its measurements to $b_{k,i}$ ($i = 1, \ldots, M_k$) bits, respectively. Denote the source throughput of sensor node k, i.e., the total amount of data in bits generated at sensor node k as S_k, and the amount of data in bits transmitted from sensor node i to sensor node j as $f_{i,j}$. According to network lifetime bound shown in (9), the network lifetime maximization problem can be formulated as a nonlinear programming (NLP) problem as follows:

$$\text{maximize} \quad D_r \left(\sum_{k=1}^{N} \sum_{i=1}^{M_k} \frac{1}{\pi_k^2(\sigma_k^2, b_{k,i})} \right) \tag{11}$$

subject to

$$\sum_{i=1, i \ne k}^{N} f_{i,k} + S_k = \sum_{j=1, j \ne k}^{N+1} f_{k,j}, \forall k \in [1, N] \tag{12}$$

$$\sum_{j=1, j \ne k}^{N+1} f_{k,j} C_{k,j} \leqslant P_k, \forall k \in [1, N] \tag{13}$$

$$S_k = \sum_{i=1}^{M_k} b_{k,i}, \forall k \in [1, N] \tag{14}$$

where

$$\begin{aligned} S_k &\ge 0, \forall k \in [1, N] \\ M_k &\ge 0, \forall k \in [1, N] \\ b_{k,i} &\ge 0, \forall k \in [1, N], i \in [1, M_k] \\ f_{i,j} &\ge 0, \forall i \in [1, N], j \in [1, N+1] \end{aligned} \tag{15}$$

where (12) and (13) represent two constraints of the optimization problem:

1) *flow conservation*: the amount of data transmitted by a sensor node is equal to the sum of the amount of data received by the sensor node and the amount of data generated by the sensor node itself;
2) *energy constraint*: the amount of data transmitted by a sensor node is limited by the energy supply of the sensor node.

It is noted that the problem given above is a nonlinear programming problem since the objective function in (11) nonlinearly depends on the variables $b_{k,i}$.

C. Separation of Source Coding Optimization With Multihop Routing Optimization

To maximize the objective function in (11), there are three factors needed to be optimized together: *i*) source coding at each sensor, i.e., quantization level $b_{k,i}$ for each observation i of each sensor k, *ii*) source throughput of each sensor, i.e., the total number of observations M_k and the total amount of data in bits S_k generated at each sensor k, and *iii*) multihop routing, i.e., the feasible network flow $\{f_{i,j} : i, j \in [1, N+1]\}$ satisfying both the flow conservation constraint in (12) and energy constraint in (13). Fortunately, the source coding optimization can be decoupled from the source throughput and multihop routing optimization as shown in Proposition 2 below.

Proposition 2: For the nonlinear programming model stated in (11)–(14), given the source throughput $\{S_k, k \in [1, N]\}$, the source coding optimization can be decoupled from multihop routing optimization.

Proof: As shown in (11), the objective function is

$$D_r \left(\sum_{k=1}^{N} \sum_{i=1}^{M_k} \frac{1}{\pi_k^2(\sigma_k^2, b_{k,i})} \right)$$

which only depends on the source throughput $S_k = \sum_{i=1}^{M_k} b_{k,i}$, ($k \in [1, N]$) and source coding scheme, but does not depend on how the source data is transmitted to the fusion center. On the other hand, the flow conservation in (12) and the energy constraint in (13) only depends on the source throughput S_k, but does not depend on the source coding. Thus, given the source throughput S_k of each sensor k, the source coding optimization is independent from the multihop routing optimization. ∎

According to the separation principle of source coding optimization with multihop routing optimization, we can solve the original optimization problem stated in (11)–(14) in two steps without loss of optimality: *i*) optimizing the source coding for given source throughput, and *ii*) optimizing the source throughput and multihop routing jointly, based on the optimal source coding. In the next two sections, we will address these two subproblems, respectively.

IV. Source Coding Optimization

In this section, we optimize the source coding for a given source throughput S_k of each sensor $k \in [1, N]$, i.e., find the optimal quantization level $b_{k,i}$ for each observation i of each sensor k to maximize the network lifetime bound. Mathematically, the problem is formulated as follows:

$$\max\ D_r\left(\sum_{k=1}^{N}\sum_{i=1}^{M_k}\frac{1}{\pi_k^2(\sigma_k^2, b_{k,i})}\right)$$
$$s.t. \sum_{i=1}^{M_k} b_{k,i} = S_k, \quad \forall k \in [1, N] \tag{16}$$

where $M_k \geq 0$ and $b_{k,i} \geq 0$ defined as before are variables to be optimized.

A. Equivalent 1-Bit MSE Function

To facilitate the solution to (16), we first introduce a concept of equivalent 1-bit MSE function.

Definition 2 (Equivalent 1-Bit MSE Function): For a quantized message from a sensor with observation noise variance σ^2 and quantization bit rate b, the estimation variance bound is $\pi^2(\sigma^2, b) := \sigma^2 + W^2/(2^b - 1)^2$ as shown in Section II-B. Then, the equivalent 1-bit MSE function is defined as

$$g(\sigma^2, b) := b \cdot \pi^2(\sigma^2, b) = b \cdot \left(\sigma^2 + \frac{W^2}{(2^b - 1)^2}\right). \tag{17}$$

From (17), we see that $\pi^2(\sigma^2, b) = \left(b \cdot 1/g(\sigma^2, b)\right)^{-1}$, which means that a b-bit quantization sensor with estimation MSE $\pi^2(\sigma^2, b)$ can be treated as b equivalent 1-bit sensors, each with the same estimation MSE $g(\sigma^2, b)$. That is why $g(\sigma^2, b)$ is called equivalent 1-bit MSE function.

It is easy to show that $g(\sigma^2, b)$ is convex over $b > 0$. Then, we further define the optimal 1-bit MSE function $g^{\rm opt}(\sigma^2)$ and the corresponding optimal quantization bit rate $b^{\rm opt}(\sigma^2)$ as follows:

$$b^{\rm opt}(\sigma^2) = \arg\min_{b \in \mathbb{Z}^+} g(\sigma^2, b)$$
$$g^{\rm opt}(\sigma^2) = \min_{b \in \mathbb{Z}^+} g(\sigma^2, b) = g(\sigma^2, b^{\rm opt}(\sigma^2)) \tag{18}$$

where the minimization involves just a simple one-dimensional numerical search. Note that $b \in \mathbb{Z}^+$ in (18) since the quantization bit rate must be integer in practice. Fig. 2 shows the optimal quantization bit rate $b^{\rm opt}(\sigma^2)$ verse different signal to noise ratio (SNR) defined as $SNR = 10\log_{10}(W^2/\sigma^2)$. As shown in Fig. 2, the optimal quantization bit rate increases over SNR, i.e., decreases over the observation noise σ^2. Also, as shown in Lemma 1 in Appendix B, the optimal 1-bit MSE function $g^{\rm opt}(\sigma^2)$ increases over the observation noise variance σ^2.

B. Upper Bound of Network Lifetime

Based on the definitions above, the network lifetime bound for estimation can be reformulated as a linear function of the source throughput S_k $(k = 1, \ldots, N)$ as shown in Theorem 1.

Theorem 1: Given the source throughput S_k of all sensor nodes $k \in [1, N]$ and the estimation distortion requirement D_r, the bound of function-based network lifetime for estimation is

$$L \leq D_r\left(\sum_{k=1}^{N}\frac{S_k}{g_k^{\rm opt}(\sigma_k^2)}\right) \tag{19}$$

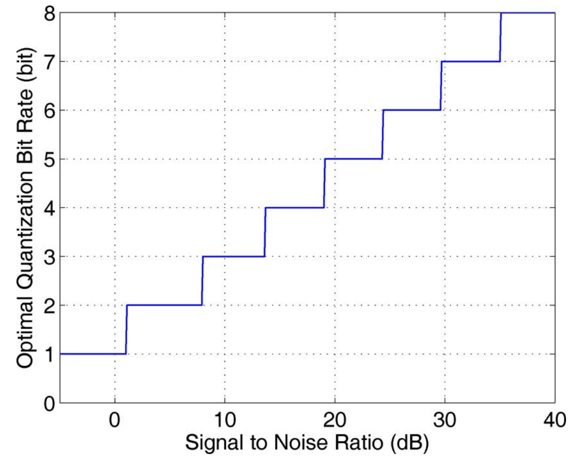

Fig. 2. Optimal quantization bit rate versus SNR.

where $g_k^{\rm opt}(\sigma_k^2)$ is the optimal 1-bit MSE function of sensor node k.

Proof: Assume sensor $k \in [1, N]$ makes a total of M_k measurements, each with quantization bit rate $b_{k,i}$, respectively, such that $\sum_{i=1}^{M_k} b_{k,i} \leq S_k$. Then as shown in (9), the network lifetime bound is

$$L \leq D_r\left(\sum_{k=1}^{N}\sum_{i=1}^{M_k}\frac{1}{\pi_k^2(\sigma_k^2, b_{k,i})}\right). \tag{20}$$

According to the definition of $g(\sigma^2, b)$ and $g^{\rm opt}(\sigma^2)$ in (17), (18) and the source throughput constraints in (16),

$$\begin{aligned} L &\leq D_r\left(\sum_{k=1}^{N}\sum_{i=1}^{M_k}\frac{1}{\pi_k^2(\sigma_k^2, b_{k,i})}\right) \\ &= D_r\left(\sum_{k=1}^{N}\sum_{i=1}^{M_k}\frac{b_{k,i}}{g_k(\sigma_k^2, b_{k,i})}\right) \\ &\leq D_r\left(\sum_{k=1}^{N}\sum_{i=1}^{M_k}\frac{b_{k,i}}{g_k^{\rm opt}(\sigma_k^2)}\right) \\ &\leq D_r\left(\sum_{k=1}^{N}\frac{S_k}{g_k^{\rm opt}(\sigma_k^2)}\right) \end{aligned} \tag{21}$$

thus, the theorem is proved. ■

Note that the equality in (21) is achieved when each sensor node adopts optimal source coding, i.e., optimal quantization bit rate $b^{\rm opt}(\sigma^2)$ to quantize its observations. As shown before, the optimal quantization bit rate $b^{\rm opt}(\sigma^2)$ of each sensor can be easily obtained by minimizing its equivalent 1-bit MSE function, which only depends on its own observation noise variance, therefore, this optimization can be done in a distributed manner. It is also worth noting that the optimal source coding is independent from the source throughput, while the source throughput at each sensor determines the total number of observations the sensor makes.

V. Joint Optimization of Source Throughput and Multihop Routing

As shown in (19) in Theorem 1, the network lifetime bound depends on the source throughput S_k for all sensors $k \in [1, N]$, which are unknown variables to be optimized. In multihop wireless sensor networks, each sensor not only transmits the data generated by itself, but also relays the data for other sensors. Since the total amount of data each sensor can transmit and relay is limited by the energy supply of the sensor node, the source throughput of each sensor and the multihop routing path from each sensor to the sink node need to be optimized together.

A. Linear Programming (LP) Formulation

As shown in Theorem 1, the nonlinear objective function in (11) can be reformulated as a linear function of the source throughput S_k ($k \in [1, N]$) by the optimal source coding, then the original network lifetime bound maximization problem shown in Section III-B can be reformulated as a linear programming (LP) problem as follows:

$$\text{maximize} \quad D_r \left(\sum_{k=1}^{N} \frac{S_k}{g_k^{\text{opt}}(\sigma_k^2)} \right) \tag{22}$$

subject to

$$\sum_{i=1, i\neq k}^{N} f_{i,k} + S_k = \sum_{j=1, j\neq k}^{N+1} f_{k,j}, \quad \forall k \in [1, N] \tag{23}$$

$$\sum_{j=1, j\neq k}^{N+1} f_{k,j} C_{k,j} \leqslant P_k, \quad \forall k \in [1, N] \tag{24}$$

where

$$S_k \geq 0, \forall k \in [1, N]$$
$$f_{i,j} \geq 0, \forall i \in [1, N], j \in [1, N+1] \tag{25}$$

and all variables are defined as before.

In summary, the network lifetime bound maximization for estimation can be formulated as a linear programming problem as shown in (22)–(24), which can be easily solved using any LP solver, such as [43] used in our simulations.

The linear programming problem in (22)–(24) can be understood as a *weighted data gathering* problem since the objective function in (22) is the weighted sum of the amount of data generated at all sensors, where the weight of the data from sensor k ($k = 1, \ldots, N$) is the inverse of its corresponding optimal 1-bit MSE function $g_k^{\text{opt}}(\sigma_k^2)$. As shown in Lemma 1, $g^{\text{opt}}(\sigma^2)$ increases over σ^2, so the weight decreases over σ^2, that is to say, it is more desirable to get data from the sensor nodes with smaller observation noise. It is also noted that if some sensors in the networks can act only as a relay, i.e., no observation capabilities, the linear programming model above still works by simply setting the weights of the data from the relay-only sensors as 0.

B. Character-Based Routing

Though the multihop routing path for the weighted data gathering problem can be easily obtained by solving the associated linear programming problem using any LP solver, it is interesting to note that, in the optimal multihop routing structure for this problem, a sensor node only relays data generated by sensor nodes with higher importance, i.e., bigger weight, as shown in Theorem 2. That is to say, the optimal routing is based on the character (fidelity and importance) of the sensor nodes, thus it is called *character-based routing*. Character-based routing is a new notion for routing and it is different from the traditional distance-based routing, such as shortest path tree, where a sensor node closer to the sink node relays information for sensor nodes farther away from the sink node.

Theorem 2: The optimal routing structure for the weighted data gathering problem shown in (22)–(24) is character-based routing, where a sensor node only relays data generated by sensor nodes with higher importance, i.e., bigger weight. More specifically, in the optimal flow and routing solution, let η be a subflow with data volume S, generated at sensor i_0 and relayed by sensors $i_1, \ldots, i_T$ sequentially to the fusion center, i.e.

$$S_{i_0}^{\eta} = f_{i_0,i_1}^{\eta} = f_{i_1,i_2}^{\eta} = \cdots = f_{i_{T-1},i_T}^{\eta} = f_{i_T,N+1}^{\eta} = S \tag{26}$$

then,

$$\sigma_{i_0}^2 \leq \sigma_{i_t}^2, \forall t \in [1, T]. \tag{27}$$

Proof: Refer to Appendix B for the complete proof of this statement. ∎

C. Special Case: Homogeneous Networks

In homogeneous wireless sensor networks, where each sensor has the same observation noise variance, i.e., $\sigma_k^2 = \sigma^2$ ($k = 1, \ldots, N$), single-hop routing path, i.e., all sensors transmit their observations to the fusion center directly, can maximize the weighted data gathering as shown in Proposition 3 below. Furthermore, the network lifetime bound for estimation can be easily quantified as shown in Proposition 4 below.

Proposition 3: In a homogeneous network with N sensors and observation noise variance σ^2, single-hop routing can maximize the weighted data gathering as in (22)–(24).

Proof: Refer to Appendix C for the complete proof of this statement. ∎

Proposition 4: In a homogeneous network with N sensors and observation noise variance σ^2, denote the energy supply of sensor k ($k = 1, \ldots, N$) as P_k, then the network lifetime bound for estimation is

$$L \leq D_r \left(\sum_{k=1}^{N} \frac{P_k}{C_{k,N+1} \cdot g^{\text{opt}}(\sigma^2)} \right) \tag{28}$$

where D_r is the estimation distortion requirement, and $C_{k,N+1}$ defined as in (10) denotes the energy cost for sensor k to transmit 1-bit message to the fusion center directly.

Proof: As shown in Proposition 3, single-hop routing can maximize weighted data gathering, thus the network lifetime bound for estimation. In single-hop wireless sensor network, each sensor transmits all its measurements to the fusion center directly, and no energy is used to relay other sensors' data, thus the maximum source throughput of each sensor node is easily obtained as $S_k = P_k / C_{k,N+1}$.

Therefore, according to Theorem 1, the network lifetime bound for estimation in a homogeneous network is $L \leq D_r \left(\sum_{k=1}^{N} \left(P_k / (C_{k,N+1} \cdot g^{\text{opt}}(\sigma^2)) \right) \right)$. ■

VI. Simulation Results

In homogeneous sensor networks, the network lifetime bound for estimation is maximized by single-hop routing and optimal source coding as shown in Proposition 4, while in heterogeneous sensor networks, the network lifetime bound for estimation is maximized by optimal source coding and optimal multihop routing jointly. To demonstrate the performances of the optimal coding scheme in Section IV and the optimal multihop routing in Section V, we simulate a homogeneous sensor network and a heterogeneous sensor network, respectively.

A. Homogeneous Sensor Networks

In this section, we simulate a homogeneous sensor network with $N = 500$ sensors, where the noise variance σ_k^2, the initial energy supply P_k, and the distance to the fusion center d_k for any sensor k is the same. Without loss of generality, we assume the range of the observation signal is $[-1, 1]$, i.e., $W = 1$, and path loss exponent $\alpha = 2$ (free space). Define the signal-to-noise ratio (SNR) as $\text{SNR} = 10 \log_{10}(W^2/\sigma^2)$ and generate different SNR by changing the observation noise variance σ^2. In order to demonstrate the efficiency of the proposed optimal source coding scheme, we compare the proposed algorithm with a heuristic method, where each sensor uses the same amount of energy at each estimation task period, thus all the sensors will deplete at the same time.

Denote the estimation MSE of the clairvoyant estimator as $D_0 = \left(\sum_{k=1}^{N} (1/\sigma_k^2) \right)^{-1}$ and define the normalized estimation MSE requirement as $D_n = D_r / D_0$. Fig. 3 shows the ratio of network lifetime bound achieved by the proposed algorithm to that by the heuristic method under different SNRs and different normalized estimation MSE requirements. From Fig. 3, we can see that a significant gain is achieved by the proposed algorithm compared with heuristic method, and the gain increases with normalized estimation MSE requirement increasing because the energy is less optimally used by the heuristic method when the normalized estimation MSE requirement increases. It is also noted that the gain is bigger for higher SNR.

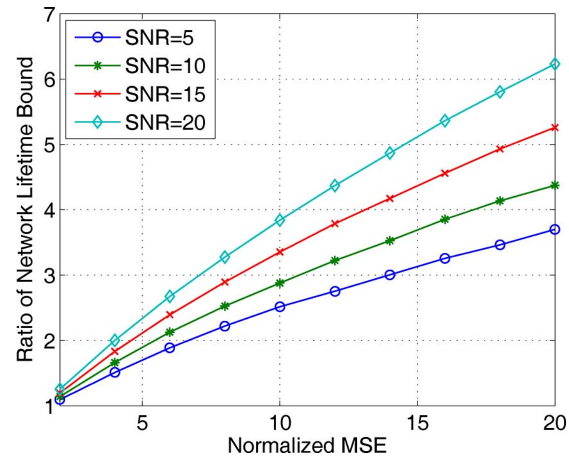

Fig. 3. Ratio of network lifetime bound for homogeneous sensor networks under different SNRs and normalized estimation MSE requirements.

B. Heterogeneous Sensor Networks

In this section, we simulate a heterogeneous sensor network with N sensors, where the observation noise variance of each sensor is assumed to be

$$\sigma_k^2 = \beta + \gamma z_k, k = 1, \ldots, N \tag{29}$$

where β models the network-wide noise variance threshold, γ controls the underlying variation from sensor to sensor, and $z_k \sim \chi_1^2$ is a Chi-Square distributed random variable with one degree of freedom. It is noted that the network is homogeneous for the special case of $\gamma = 0$. In the experiments, we assume $\beta = 0.01$ and $\gamma = 0.00, 0.05, 0.10, 0.15,$ or 0.20. Assume all sensors are independently and uniformly distributed in a rectangular region of $[-5, 5, -5, 5]$, and the fusion center is located at the central point of the region, i.e., (0, 0). And the initial energy is still assumed to be the same for all sensors. Assume the estimation MSE requirement is $D_r = 5D_0$.

For a given network setting, the optimal source coding and optimal multihop routing solutions are determined to maximize the network lifetime bound for estimation. To demonstrate the efficiency of the proposed algorithms, we compare it with two heuristic methods.

1) *Heuristic-I*: single-hop routing with uniform energy scheduling for each sensor.
2) *Heuristic-II*: single-hop routing with optimal source coding and energy scheduling.

Fig. 4(a) and (b) shows the ratio of network lifetime bound achieved by the proposed algorithm to that by the *Heuristic-I* and *Heuristic-II* methods under different total number of sensors and different sensor noise variation parameters γ, respectively, where all the simulation results are obtained by repeating the experiments for 2000 times and averaging the individual results. From Fig. 4(a) and (b), we can see that the proposed algorithms improve the network lifetime bound significantly compared with both *Heuristic-I* and *Heuristic-II* methods, and the gain becomes more significant when the sensor network becomes denser or the observation noise variances become more diverse, i.e., γ becomes bigger. It is also worth noting that the similar conclusions can be drawn for different estimation MSE requirements D_r except that the actual value in Fig. 4(a) will be even bigger with bigger D_r as we have shown in Fig. 3.

It is noted that both the optimal method and the *Heuristic-II* method use optimal source coding, and the only difference is that optimal multihop routing is used by the optimal solution, while single-hop routing is used by the *Heuristic-II* method. From Fig. 4(b), we see that the *Heuristic-II* method is also optimal when $\gamma = 0.00$, which confirms our conclusion in Proposition 3 that single-hop routing can maximize the network lifetime bound for homogeneous networks. From Fig. 4(b), we also can see that optimal multihop routing improves the network lifetime bound significantly compared with single-hop routing for

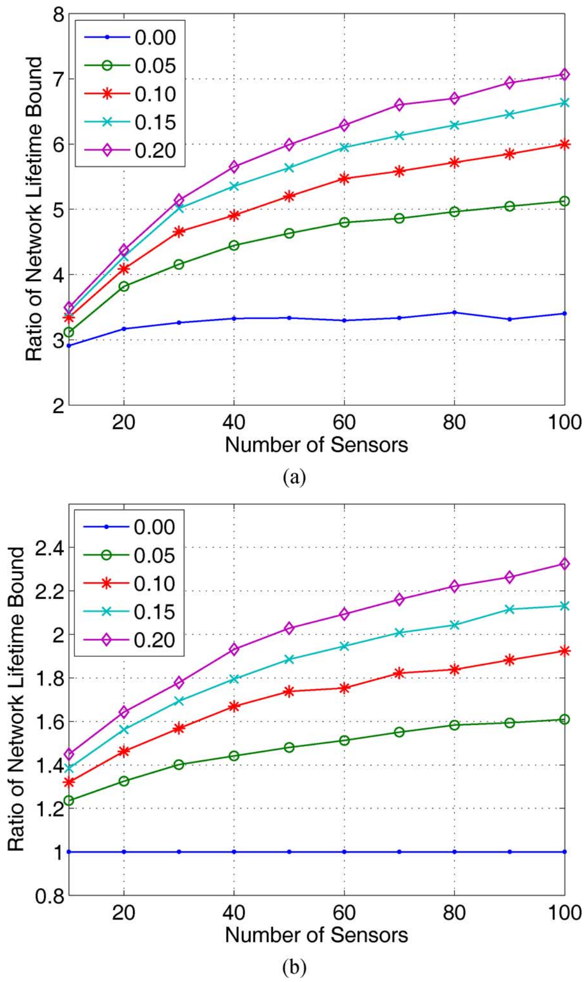

Fig. 4. Ratio of network lifetime bound by the proposed algorithm to that by two heuristic methods under different total number of sensors and different sensor noise variation parameters ($\gamma = 0.00, 0.05, 0.10, 0.15, 0.20$): (a) compared with *Heuristic-I*, (b) compared with *Heuristic-II*.

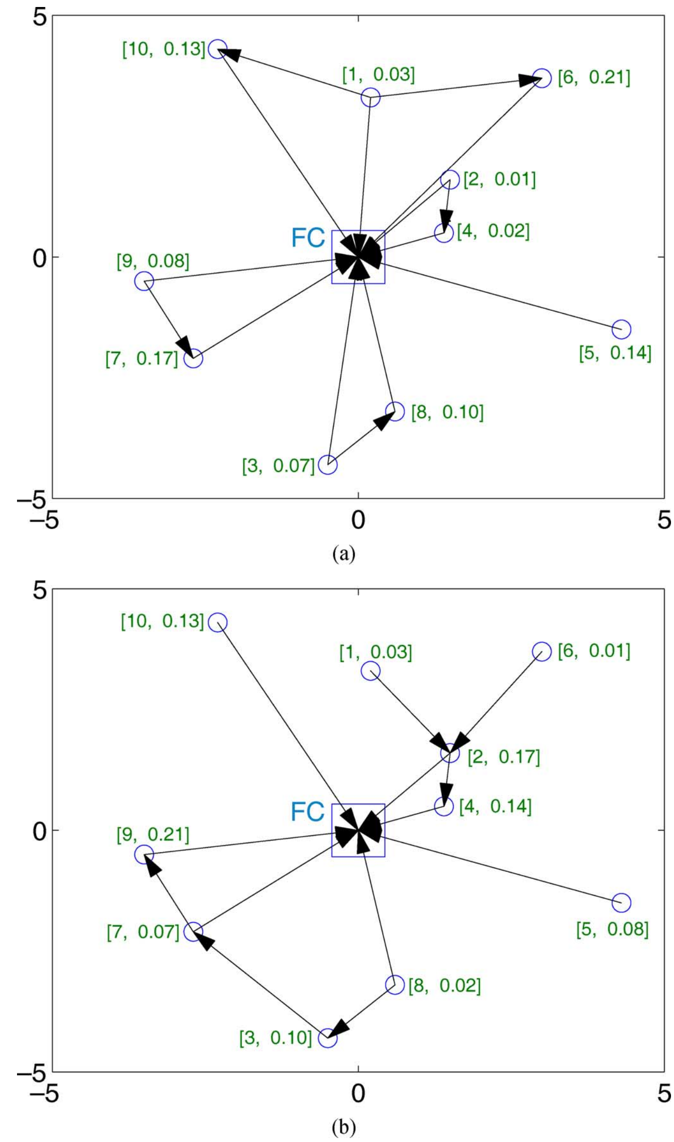

Fig. 5. The optimal multihop routing path for two example heterogeneous sensor networks with same sensor locations but different observation noise variances.

heterogeneous networks. Furthermore, the gain is more significant when the network is denser since there are more opportunities for multihop routing. Also the gain is more significant when the observation noise variances are more diverse since the optimal multihop routing is character-based as shown in Section V-B.

To further demonstrate the character-based routing, Fig. 5(a) and (b) shows two example heterogeneous sensor networks with $N = 10$ sensor nodes, where each circle denotes a sensor node. There are two numbers in the brackets around each sensor node, where the first one denotes its index and the second one denotes its observation noise variance. In these two networks, the sensor locations are the same, while the observation noise variances are different. From Fig. 5(a) and (b), we can see that the optimal routing completely changed due to the different observation noise variances, and the sensors only relay information generated at other sensors with smaller observation noise variance, such as in Fig. 5(a), sensor 8 relays information from sensor 3, while in Fig. 5(b), sensor 3 relays information from sensor 8 even though sensor 3 is farther away from the fusion center than sensor 8. The intuitive explanation is that sensor 8 has a very small observation noise variance, then it is desirable to gather as much data as possible from sensor 8, thus sensor 8 should transmit its data to its nearest neighbor (sensor 3) if possible to save transmission energy and improve source throughput. It is also noted that in Fig. 5(b), sensor 7 relays some information from sensor 3 even though sensor 7 has smaller observation noises than sensor 3 because the relayed information is originally generated at sensor 8 other than sensor 3.

VII. Conclusion

In this paper, we consider the distributed estimation in energy-limited wireless sensor networks from *lifetime-distortion* perspective, which is rarely addressed in the literature. From the application aspect, we are interested in the estimation task cycles the network can accomplish before the network becomes nonfunctional other than whether any individual sensor node is dead, thus we introduce a concept of function-based network lifetime. Based on this concept, it is shown that the network lifetime bound maximization problem can be formulated as a nonlinear programming (NLP) problem, where there are three factors needed to be optimized together: *i*) source coding at each sensor, i.e., quantization level for each observation, *ii*) source throughput of each sensor, and *iii*) multihop routing. We further show that the source coding can be optimized independently from the source throughput and multihop routing, and the optimal source coding is achieved by maximizing the equivalent 1-bit MSE function. Then based on the optimal source coding, the nonlinear programming problem of network lifetime bound maximization can be reformulated as a linear programming (LP) problem, which can be easily solved by any LP solver.

On the other hand, the linear programming formulation for network lifetime bound maximization problem can be understood as a *weighted data gathering* problem, where the objective is to maximize the weighted sum of the amount of data generated at all sensors. The weight of each sensor is inversely proportional to its observation noise variance, which is meaningful since the data from sensors with smaller noise variance is more useful. Furthermore, we find out that the optimal routing solution is *character-based routing*, where a sensor node only relays data from sensor nodes with smaller observation noise variance. Different from the traditional distance-based routing, where the routing path is selected based on the distance to the destination, *character-based routing* explicitly takes into account the information character in the routing selection.

Further generalization of the new notions of function-based network lifetime and character-based routing is an interesting direction for the future work. A distributed implementation of the character-based multihop routing to achieve the maximum network lifetime bound and some simple heuristic algorithms to achieve the close-to-optimal performance are also interesting directions we plan to undertake in the future. To facilitate the problem, we have assumed in this paper that the observation noises among different sensors are uncorrelated and the channels from the local sensors to the fusion center are error free. As the future work, we also plan to relax the above assumptions and study the more general distributed parameter estimation problems.

Appendix A
Proof of Proposition 1

Assume a sensor network with N sensors, each with observation noise variance σ_k^2. Assume sensor k ($k = 1,\ldots,N$) can make a total of M_k measurements and quantize its measurements using probabilistic quantization scheme to $b_{k,i}$ ($i = 1,\ldots,M_k$) bits, respectively, before it depletes. To satisfy the given estimation distortion requirement D_r, at each estimation cycle, a subset of sensors are required to observe the parameter θ and transmit their quantized measurements to the fusion center to make the final estimation.

Assume the network lifetime for this network is L. At each estimation cycle $l \in [1, L]$, denote the subset of observations each sensor k makes and sends to the fusion center is $O_{k,l}$. Then for any sensor $k \in [1, N]$, we have

$$\begin{aligned} &O_{k,i} \cap O_{k,j} = \emptyset, \forall i,j \in [1,L], \text{ and } i \neq j \\ &\bigcup_{l=1}^{L} O_{k,l} \subseteq \{1,\ldots,M_k\} \end{aligned} \tag{30}$$

and for any estimation cycle $l \in [1, L]$, we have

$$\left(\sum_{k=1}^{N}\sum_{i\in O_{k,l}} \frac{1}{\pi_k^2(\sigma_k^2, b_{k,i})}\right)^{-1} \leq D_r. \tag{31}$$

So

$$\sum_{l=1}^{L}\sum_{k=1}^{N}\sum_{i\in O_{k,l}} \frac{1}{\pi_k^2(\sigma_k^2, b_{k,i})} \geq \frac{L}{D_r} \tag{32}$$

i.e.

$$\sum_{k=1}^{N}\sum_{i=1}^{M_k} \frac{1}{\pi_k^2(\sigma_k^2, b_{k,i})} \geq \frac{L}{D_r} \tag{33}$$

therefore

$$L \leq D_r\left(\sum_{k=1}^{N}\sum_{i=1}^{M_k} \frac{1}{\pi_k^2(\sigma_k^2, b_{k,i})}\right). \tag{34}$$

Appendix B
Proof of Theorem 2

Lemma 1: For two sensors with different observation noise variances σ_1^2 and σ_2^2, if $\sigma_1^2 < \sigma_2^2$, then

$$g^{\rm opt}(\sigma_1^2) < g^{\rm opt}(\sigma_2^2), \tag{35}$$

where, $g^{\rm opt}(\sigma^2)$ is the optimal 1-bit MSE function defined in (18).

Proof: According to the definition in (17), $g(\sigma^2, b)$ is an increasing function over σ^2, i.e.

$$g(\sigma_1^2, b) < g(\sigma_2^2, b). \tag{36}$$

According to the definition of $b^{\rm opt}(\sigma^2)$ and $g^{\rm opt}(\sigma^2)$, we have that

$$\begin{aligned} g^{\rm opt}(\sigma_1^2) &= g(\sigma_1^2, b^{\rm opt}(\sigma_1^2)) \\ &\leq g(\sigma_1^2, b^{\rm opt}(\sigma_2^2)) \\ &< g(\sigma_2^2, b^{\rm opt}(\sigma_2^2)) \\ &= g^{\rm opt}(\sigma_2^2). \end{aligned} \tag{37}$$

■

Now, we begin to prove Theorem 2 by contradiction. Assume a sensor i_m ($m \in [1, T]$) on the routing path of the subflow η has a smaller observation noise variance than the source node i_0, i.e.,

$$\sigma_{i_m}^2 < \sigma_{i_0}^2 \tag{38}$$

then, remove the subflow η and add a new subflow ξ with the same data volume, which is generated at sensor node i_m and transmitted to the fusion center through sensor nodes $i_{m+1},\ldots,i_T$ sequentially, i.e.

$$\begin{aligned} S_{i_0}^{\eta} &= f_{i_0,i_1}^{\eta} = f_{i_1,i_2}^{\eta} = \cdots = f_{i_{T-1},i_T}^{\eta} = f_{i_T,N+1}^{\eta} = 0, \\ S_{i_m}^{\xi} &= f_{i_m,i_{m+1}}^{\xi} = \cdots = f_{i_{T-1},i_T}^{\xi} = f_{i_T,N+1}^{\xi} = S. \end{aligned} \tag{39}$$

First, it is easy to show that the new flow is feasible, that is to say, it satisfies the flow conservation and energy constraints as shown in (23) and (24).

Next, assume the total data volume generated at each sensor k is S_k and denote ϕ_0 and ϕ_1 as the objective function divided by D_r before or after removing the subflow η and adding the new subflow ξ, i.e.

$$\phi_0 = \sum_{k=1}^{N} \frac{S_k}{g_k^{\text{opt}}(\sigma_k^2)}$$
$$\phi_1 = \sum_{k=1,k\neq i_0,i_m}^{N} \frac{S_k}{g_k^{\text{opt}}(\sigma_k^2)} + \frac{S_{i_0} - S}{g_{i_0}^{\text{opt}}(\sigma_{i_0}^2)} + \frac{S_{i_m} + S}{g_{i_m}^{\text{opt}}(\sigma_{i_m}^2)} \quad (40)$$

then

$$\phi_1 - \phi_0 = \frac{S}{g_{i_m}^{\text{opt}}(\sigma_{i_m}^2)} - \frac{S}{g_{i_0}^{\text{opt}}(\sigma_{i_0}^2)} > 0 \quad (41)$$

because $g_{i_m}^{\text{opt}}(\sigma_{i_m}^2) < g_{i_0}^{\text{opt}}(\sigma_{i_0}^2)$ when $\sigma_{i_m}^2 < \sigma_{i_0}^2$ as shown in Lemma 1. It means that the objective function in (22) is increased by removing the subflow η and adding the new subflow ξ, which contradicts with the optimality of the original flow and routing solution. So the assumption made in (38) does not hold, therefore, the Theorem is proved.

APPENDIX C
PROOF OF PROPOSITION 3

In the optimal flow and routing solution for the weighted data gathering problem in homogeneous wireless sensor networks, assume there is a multihop subflow η with data volume S, generated at sensor i_0 and transmitted to the fusion center through sensors $i_1, \ldots, i_T$ sequentially, i.e.

$$S_{i_0}^{\eta} = f_{i_0,i_1}^{\eta} = f_{i_1,i_2}^{\eta} = \cdots = f_{i_{T-1},i_T}^{\eta} = f_{i_T,N+1}^{\eta} = S \quad (42)$$

then, remove this multihop subflow η and add a serial of single-hop subflow $\xi_0, \ldots, \xi_T$ as follows:

$$S_{i_t}^{\xi_t} = f_{i_t,N+1}^{\xi_t} = \frac{C_{i_t,i_{t+1}}}{C_{i_t,N+1}} \cdot S, \forall t \in [0, T-1]$$
$$S_{i_T}^{\xi_T} = f_{i_T,N+1}^{\xi_T} = S. \quad (43)$$

Similar with the proof for Theorem 2, it is easy to show that both the flow conservation and energy constraints as shown in (23) and (24) are satisfied by removing the subflow η and adding the new subflows $\xi_0, \ldots, \xi_T$.

Next, assume the total data volume generated at each sensor k is S_k and denote ϕ_0 and ϕ_1 as the objective function divided by D_r before or after removing the subflow η and adding the new subflows $\xi_0, \ldots, \xi_T$, i.e.

$$\phi_0 = \frac{1}{g^{\text{opt}}(\sigma^2)} \sum_{k=1}^{N} S_k$$
$$\phi_1 = \frac{1}{g^{\text{opt}}(\sigma^2)} \left(\sum_{k=1,k\neq i_0,i_T}^{N} S_k + (S_{i_0} - S) + (S_{i_T} + S) \right) + \frac{1}{g^{\text{opt}}(\sigma^2)} \sum_{t=0}^{T-1} \frac{C_{i_t,i_{t+1}}}{C_{i_t,N+1}} \cdot S \quad (44)$$

then

$$\phi_1 - \phi_0 = \frac{1}{g^{\text{opt}}(\sigma^2)} \sum_{t=0}^{T-1} \frac{C_{i_t,i_{t+1}}}{C_{i_t,N+1}} \cdot S \geq 0 \quad (45)$$

where, the equality holds only when the fusion center is not in the transmission range of all sensors $i_0, \ldots, i_{T-1}$, i.e., $C_{i_t,N+1} = \infty$ for all $t \in [0, T-1]$, otherwise, $\phi_1 - \phi_0 > 0$. It means that for homogeneous networks with unlimited transmission range for each sensor, single-hop routing can gather greater amount of data than multihop routing, while for homogeneous networks with limited transmission range for each sensor, single-hop routing can gather no less amount of data than multihop routing.

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